BORROWING INTEREST RATE AS A FUNCTION OF DEBT-EQUITY RATIO IN CAPITAL BUDGETING MODELS

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BORROWING INTEREST RATE AS A FUNCTION OF DEBT-EQUITY RATIO IN CAPITAL BUDGETING MODELS

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SUMMARY

Most of the research on mathematical programming techniques in capital budgeting models consider the borrowing interest rate as a constant rate with fixed debt limits, or by establishing a rising supply curve for funds. It is not considered in those models that the lender of funds take into account, among other factors, the capital structure of the firm in determining its borrowing ability. The objective of this work is to extend a capital budgeting model, taking Weingartner's basic horizon model as a point of departure, including the borrowing interest rate as a function of the debt-equity ratio, to develop a computational algorithm for solving the extended model, and to make economic interpretations.

In determining the interest rate--debt/equity ratio relationship, there were used publicly available data for two industries, and three options were used in expressing the debt-equity ratio: market value of debt over market value of equity, book value of debt over book value of equity, and book value of debt over market value of equity.

Three models were developed. The first and the most general is a non-convex nonlinear programming problem, which was solved by the Hooke and Jeeves algorithm. Different starting points were used, resulting in essentially similar

solutions. Thus, the nonconvexity of the model does not appear to result in different optima, at least in the range of practical values.

The second model is a convex nonlinear programming model. It differs from the general model by the assumption of a fixed equity. It was solved also by the Hooke and Jeeves algorithm.

The third model, an iterative linear model, can be used as a planning tool where the assumption of an average interest rate is appropriate. This approach recasts the nonlinear problem as a LP problem with fixed interest rates, and then makes an iterative process adjusting the interest rates until they correspond with the resulting debt-equity ratio.

The Hooke and Jeeves algorithm is not efficient, but for this type of problem, it seems capable of finding the solution, whereas other methods were attempted with unsatisfactory results.

In general, it is concluded that either the non-convex model or the iterative linear model can be used to represent the relationship between borrowing interest rate and debt-equity ratio. The non-convex model includes explicit interest rate functions, and is amendable to a variety of solution techniques. The models presented assume annual debt instruments and thus predict much more rapid changes in aggregate interest payments than would actually occur; the use of the LP model, which employs a weighted average interest rate, seems more suitable for such slower changes in actual rates.

CHAPTER I

INTRODUCTION

The Capital Budgeting Problem

The typical business firm has a set of human and economic resources which are employed in the achievement of its objectives. Almost always the firm has many opportunities to invest but has limited resources to undertake them. Such opportunities represent the use of those resources now, with the expectation of yielding a return after a certain period of time. Management has to decide how much of those scarce resources should be allocated to the various opportunities in order to maximize the benefits for the owners.

The capital budgeting problem is defined as the decision problem whereby a firm attempts to select investment projects to maximize some overall measure of benefit while meeting certain constraints on money and other resources needed to operate the projects, and meeting constraints on project interdependencies.

The capital budgeting decision affects the firm not only from the financial point of view, but in many other aspects as well. The stockholder's viewpoint is to prefer those projects which make the market value of the firm increase, so that individual share price is increased. The capital budgeting decision can impact the expectations about the income or earnings per share currently and in the future in such a way that a bad capital budgeting decision may discredit the firm's image and thus cause a loss of goodwill that could imply mistrust about the company's stability and its dividends. On the other hand, the achievement of expected returns implies the ability of the firm to fulfill its promises and the ability to make profitable future investments.

The capital budgeting decision involves many factors to be taken into consideration: the available amounts of scarce resources, the lifetimes of the proposals, the option of borrowing, the possibility of accepting fractions of proposals, the option of reinvestment for cash receipts and the capital structure of the firm, among others.

Payback methods as budgeting techniques have been declining in popularity through the years due to the use of discounted cash flow methods. Along with evaluation methods the use of optimization has become popular in solving capital budgeting problems.[14] However, these techniques have shortcomings that restrict their use to specific problems where certain assumptions must be made. There have been theoretical advances made in this field, but for the most part these have little immediate practical use because of certain assumptions as perfect capital markets, no differential tax effects, etc.[17]

Capital Budgeting Models

One of the most important contributions to capital budgeting models is Weingartner's work on the use of mathematical programming.[24] Before his study, analysis techniques generally assumed unlimited availability of capital, the possibility of borrowing and lending at a market interest rate, and usually neglected many other real world constraints. Weingartner makes use of linear and integer programming in searching for the optimal solution without an explict enumeration of all the investment combinations. He includes the possibility of borrowing and lending, borrowing limits, and contingency relationships. He derives interesting relationships using the dual variables and shows basic shortcomings of previous modeling efforts.

Since Weingartner many other contributions have appeared in the literature. Most of the formulations deal with the interest rate on borrowed money assuming a constant rate, perhaps with fixed debt limits, or by establishing a rising supply curve for funds where higher interest rates are associated with successive amounts borrowed. However, in determining borrowing ability and interest rates a lender considers, among other factors, the capital structure of the borrowing firm. The company, on the other hand, must determine not only how much capital to allocate to which proposals in order to maximize its objective, but to carry out that optimization has to consider how these proposals should be financed. Two basic financing options are long-term debt and equity. Basically, the capital structure is determined by the mixture of long-term debt and equity used by the firm to finance its operations. Any significant variation in the capital structure will cause a change in the ability to borrow funds.

The debt-equity ratio represents a trustworthy measure of the capital structure of a firm. The higher this ratio, the higher the firm's financial leverage which suggests a higher interest rate on borrowed money.

Objective of This Study

The objective of this research is to extend a capital budgeting model to include borrowing interest rate as a function of the debt-equity ratio, to demonstrate the computational ability of an algorithm for solving the extended model, and to obtain economic interpretations.

Method of Approach

The first part of the work deals with the interest rate--debt-equity relationship. Here there were used publicly available data for two important industry groups: chemicals and multi-industry companies with chemical process operations. In considering the debt-equity ratio there were followed three options: express the ratio as market value of debt over market value of equity, as book value of debt over market value of equity, and as book value of debt over book value of equity.

For the first option it is considered that when a firm obtains additional financing the cost of this financing depends on the price at which the financial instrument is sold. In the same manner, suppliers of funds have more ability to view the firm's financial position according to its financial structure as measured in the market place. As Gitman[10] establishes: "This belief is based on the premise that if the firm were to obtain its existing financing today, receiving market prices, its financial structure would be that reflected by the market--not book values of its existing financial instruments." The second option is based on the fact that the book value of debt is what the firm really owes. The point of view of the lender is to examine the balance sheet and proforma income statement of the borrowing firm, where the interest rate will vary depending on the timing and special characteristics of the issue and lender. The last option takes into account that the stockholder's equity is not only composed of common stock but other terms for which market prices are difficult to determine, as in the case of retained earnings. The book value of equity is considered to be the sum of par value of common stock, capital surplus and retained earnings, minus the carrying value of treasury stock and the liquidation value of preferred stock.

The mathematical programming model to be used should include the borrowing interest rate--debt-equity ratio relationship, resulting in a nonlinear programming problem. The capital budgeting model taken as a base is Weingartner's horizon model.

There are two basic approaches to solve the problem:

1. Recast it as linear programming problem with fixed interest rates, and then make an iterative process adjusting the interest rates until they correspond with the resulting debt-equity ratio.
2. Apply a nonlinear algorithm directly.

It is expected that a more realistic model of capital budgeting will be obtained where the criterion to assign interest rates on borrowed money agrees more closely with what happens in real situations.

Organization of Thesis

Chapter II presents a literature survey about capital budgeting models; Chapter III describes the data collection and the results of the regression analysis to develop a relationship between borrowing interest rate and the debt-equity ratio. Chapter IV presents the formulation of the linear and nonlinear programming problems, including the optimality analysis; Chapter V presents solution procedures and computational results; and Chapter VI presents the conclusions of the thesis and recommendations for future research.

CHAPTER II

REVIEW OF THE LITERATURE

In this chapter there is presented a review of the most important contributions to the development of deterministic capital budgeting techniques. Only those works more directly related to the scope of this study are treated, so little mention is made of capital budgeting models with uncertainty. There are also described some aspects of the imperfections of the capital market considered in certain models.

Definitions

It is necessary to define some concepts related to the capital budgeting decision that will be treated throughout this study.

a. Rate of return. It is defined as the interest rate that reduces the present-worth amount of a series of receipts and disbursements to zero. For a proposal j, the rate of return i_j^* satisfies the equation.[9]

$$\sum_{t=0}^{n} F_{jt} (1 + i_j^*)^{-t} = 0 \qquad (2\text{-}1)$$

where F_{jt} is the cash flow at time t for proposal j.

b. Independent proposals. A proposal is independent when its acceptance has no effect on the acceptance of other proposals.

c. Mutually exclusive proposals. A set of proposals are mutually exclusive when the acceptance of one of them will cause the rejection of the other alternatives in the set.

d. Contingent proposals. A proposal is contingent when its acceptance depends on the acceptance of other proposals.

Capital Budgeting Criteria

For a long time one of the most widely used methods in capital budgeting has been the payback method. It indicates the number of years required to recover the initial cash investment, without interest. It is defined as the ratio of the initial fixed investment over the annual cash inflows for the recovery period. If the payback period is less than some upper bound payback period, the proposal is accepted; otherwise, it is rejected. This method cannot be considered as a measure of profitability because it does not take into account cash flows after the payback period. In addition, it does not consider the magnitude and timing of cash flows during the payback period. Some authors [2,4] treat it as a constraint to be satisfied rather than a measure of profitability. The payback period method analyzes neither budget restrictions nor the way of financing the proposals, and neglects many other aspects related to the capital budgeting decision.

As reported by Klammer,[14] during the period from 1959 to 1972 the use of internal rate of return and the use of net present worth have increased by a factor of five; meanwhile the use of payback period remained constant.

The present worth criterion can be applied to independent and to mutually exclusive proposals. The procedure is to rank the investment combinations in increasing order of initial investment, calculate the present worth of the incremental investment at a rate of return specified by firm's top management, and accept those for which the incremental investment yields a positive present worth.

Joel Dean[5] established a capital rationing theory based on internal rate of return. The point where the demand schedule for capital expenditures intersects the supply curve of funds determines a cut-off rate for a specific period of time. Any proposal with a rate of return higher than this cut-off rate should be accepted. The demand schedule is built ranking the proposals in decreasing order of internal rate of return, showing the cumulative amount of money that can be invested in the proposals (investment opportunity curve). It is assumed that the objective of the firm is to maximize profits, that all opportunities for investment are perfectly known, that investments can be considered to be in the same risk class, and that the firm has access to the capital market. Although he does not consider the possibility of fixed budgets, restrictions on borrowing, the possibility of non-independent proposals, and the imperfections of the market, his work is very relevant from the point of view of establishing a systematic procedure for the capital budgeting problem.

A related procedure is presented by Gerald Fleischer in his article: "Two Major Issues Associated with the Rate of Return Method for Capital Allocation: 'The Ranking Error' and 'Preliminary Selection.'"[9] He established, in the first part of his paper, that ranking two alternatives in decreasing order of rates of return and choosing the one with the higher rate can be an incorrect decision (ranking error). He introduces the concepts: technical and financial mutual exclusiveness. A set of proposals is technically mutually exclusive when only one of the alternatives is necessary to fulfill certain functions and is financially mutually exclusive when one or more alternatives may be acceptable, but not all can be accepted because of budget limitations.

Fleischer develops a procedure for evaluating a set of proposals which can be applied to independent and/or mutually exclusive proposals. First, he defines the term "budget packages" as the set of all possible mutually exclusive feasible combinations of proposals. The maximum number of budget packages is given by

$$Q = \prod_{j=1}^{N} (M_j+1) \qquad (2\text{-}2)$$

where N is the maximum number of sets of proposals that are independent and M_j is the maximum number of proposals within each mutually exclusive set J. Then he applies an incremental analysis using the internal rate of return criterion for selecting the best package. Thuesen, Fabrycky, and Thuesen[21] extended the procedure to include contingent projects.

Lorie and Savage[16] treat the problem of allocation of resources for the case of mutually exclusive and independent proposals considering budget restrictions for two periods. They proposed an iterative process for finding the optimal solution. The method requires one to fix two constants p_1 and p_2 such that $(y-p_1c_1-p_2c_2)$ is positive, where y is the present value of the proposal and c_1 and c_2 are the present values of the net outlays required in the first and second periods, respectively. The initial valaues of p_1 and p_2 are fixed by judgment and subsequently they are changed by trial and error until the amounts to be spent in both periods are the amounts permitted by the constraints. The method attempts to give an integer solution; the values of p_1 and p_2 are closely related to Lagrange multipliers.

The Lorie-Savage problem may be generalized for n time periods. Weingartner[24] states the problem as an integer programming problem:

$$\text{maximize:} \quad \sum_{j=1}^{n} b_j x_j \qquad (2\text{-}3)$$

$$\text{subject to:} \quad \sum_{j=1}^{m} c_{tj}x_j \leq C_t \quad t = 1,2,\ldots, T \tag{2-4}$$

$$x_j \quad \text{integer} \tag{2-5}$$

where b_j = net present value of proposal j (discounted at the cost of capital of the firm, as is suggested by the authors).

c_{tj} = the outlay required for the j^{th} proposal in the t^{th} period.

C_j = the maximum permissible expenditure in period t.

$$x_j = \begin{cases} 0 \text{ if proposal j is rejected} \\ 1 \text{ if proposal j is accepted} \end{cases}$$

Another attempt for solving capital budgeting problems is presented by Baumol and Quandt.[1] They attempted, by using mathematical programming, to solve a capital budgeting problem and determine an appropriate set of discount rates simultaneously. Their model is:

$$\text{maximize:} \quad \sum_{j=1}^{n} \sum_{t=0}^{n} \left[\frac{a_{tj}}{(1+i)^t}\right] x_j \tag{2-6}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} b_{jt}x_j \leq M_t \quad t = 0,1,\ldots, h \tag{2-7}$$

$$x_j \geq 0 \quad j = 1,2,\ldots n. \tag{2-8}$$

where i = rate of interest

a_{tj} = net cash flow from a unit of project j at period t.

b_{jt} = net amount of cash used by unit of project j during period t.

x_j = number of units of project j selected.

M_t = amount of cash available from outside sources at period t.

This formulation can include mutually exclusive proposals and integer solutions. The major result of the work is that the objective function cannot be formulated until the dual problem has been solved; but the dual cannot be solved unless the optimal primal solution is known. A drawback of the model is that it does not allow for borrowing and lending between periods. To remedy these difficulties, Baumol and Quandt suggest using a linear objective function of the dividend payments:

$$\text{maximize:} \quad \sum_t U_t W_t \tag{2-9}$$

$$\text{subject to:} \quad -\sum_j a_{tj} x_j + W_t + C_t - C_{t-1} \leq M_t \tag{2-10}$$

$$x_j \geq 0 \tag{2-11}$$

W_t is the dividend at period t, U_t is the utility of a dollar at period t, and C_t is the cash amount carried from period t to t + 1. C_t can be considered as a lending activity at zero lending interest rate where no borrowing activities are allowed. From the dual problem they obtain $p_t^* \geq p_t + \overset{*}{1}$ for $t = 1,2,\ldots, T-1$, which implies that a marginal dollar at time t will always be worth at least what it would be worth one period later.

One of the most complete treatments of the use of mathematical programming to allocate capital is the work of Weingartner.[24] He defines the basic horizon model:

$$\text{maximize:} \quad \sum_{j=1}^{n} \hat{a}_j x_j + v_T - w_T \tag{2-12}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} a_{1j} x_j + v_1 - w_1 \leq D_1 \tag{2-13}$$

$$\sum_{j=1}^{n} a_{tj} x_j - (1+r) v_{t-1} + v_t + (1+r) w_{t-1} - w_t \leq D_t \qquad t = 2,3,\ldots, T \tag{2-14}$$

$$0 \leq x_j \leq 1 \quad j = 1,2,\ldots, n \tag{2-15}$$

$$w_t,\ v_t \geq 0 \quad t = 1,2,\ldots, T \tag{2-16}$$

where a_{tj} = cash flow at time t resulting from acceptance of proposal j. ($\hat{a}_{tj} > 0$ is considered as an expenditure and $\hat{a}_{tj} < 0$ is considered as a revenue).

$\hat{a}_j$ = present value at the time of the horizon of cash flows subsequent to the horizon, discounted at rate r.

D_t = amount available for investment at period t from outside sources.

w_t = amount borrowed from year t to t + 1.

v_t = amount lent from year t to t + 1.

x_j = fraction of proposal j accepted.

The objective function attempts to maximize the net value of assets at the horizon T, which are expressed as the funds available for lending and the present value of the net cash flows after the horizon. Lending and borrowing activities are permitted without limit at a specified interest rate r, and complementarity and competitiveness between proposals may be added to the established model.

Subsequently, Weingartner changes certain assumptions made under the perfect capital market conditions; there are imposed absolute limits on the amount of debt that the firm may carry. Because this type of restriction may be considered too rigid for real situations, this constraint is extended to a rising supply curve for funds. It is assumed that higher rates of interest are associated with larger amounts borrowed, due to the fact that as the debt is increased the probability of default is increased too.

Taking the concept developed above, the rising supply curve of funds can be seen as a set of absolute limits on amounts borrowed at different interest rates, obtaining a step function interpreted as the marginal cost of funds, which can be transformed into an average cost of funds curve, usually called the supply curve (Figure 2-1).

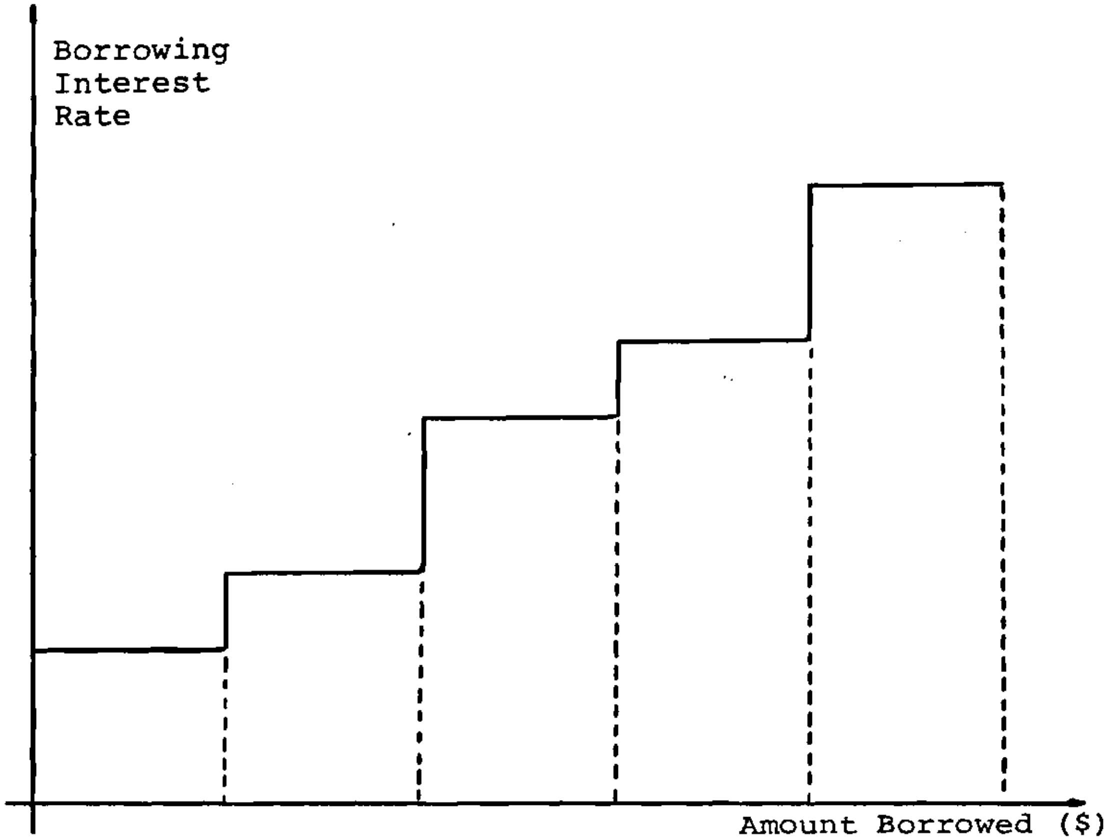


Figure 2-1. Marginal Cost of Funds Curve.

Considering r_i as the interest rate which applies to the i^{th} step of the supply curve, w_{it} as the amount borrowed in this step at year t, and B_{it} as the upper limit of the i^{th} step, the capital budgeting model becomes:

$$\text{maximize:} \quad \sum_{j=1}^{n} \hat{a}_j x_j + v_T - \sum_{i=1}^{m} w_{iT} \tag{2-17}$$

$$\text{subject to:} \quad \sum_{j=1}^{n} a_{1j} x_j + v_1 - \sum_{i=1}^{m} w_{i1} \leq D_1$$

$$t = 2,3,\ldots T. \tag{2-18}$$

$$\sum_{j=1}^{n} a_{tj} x_j - (1+r) v_{t-1} + v_t + \sum_{i=1}^{m} (1+r_i) w_{i,t-1} - \sum_{i=1}^{m} w_{it} \leq D_t \tag{2-19}$$

$$w_{it} \leq B_{it} \quad t = 1,2,\ldots, T. \quad i = 1,2,\ldots, m. \tag{2-20}$$

$$0 \leq x_j \leq 1 \quad j = 1,2,\ldots, m. \tag{2-21}$$

$$v_t, w_t \geq 0 \quad t = 1,2,\ldots, T. \tag{2-22}$$

where $r_{i-1} < r_i < r_{i+1}$. Weingartner suggests the use of integer programming in the solution of this modification of the basic horizon model. Finally, he treats the possibility of optimizing the timing and amounts of stock to be issued.

Bernhard[2] summarizes in his article: "Mathematical Programming Models for Capital Budgeting--A Survey, Generalization, and Critique," most of the most important works in the field, and proposes a generalized deterministic mathematical programming model which is essentially based on the studies of the authors discussed in his paper. He points out that the objective functions of most of the models do not allow dividend payments before or at the horizon. He defines an objective function to be maximized as $f(W_1, W_2, \ldots W_T, G)$, where W_t is the dividend paid during year t, and G is the terminal wealth, which is defined below.

The proposed model includes a cash balance restriction, as in Reference 24, adding the concept of compensating balances. There is a group payback restriction as suggested by Byrne, Charnes, Cooper, and Kortanek,[4] and also Weingartner's "manpower" restriction. There is introduced a terminal wealth definition restriction, following payment of dividends, defined as:

$$G = M' + \sum_{j=1}^{n} \hat{a}_j x_j + v_T + c_T w_T + C_T - w_T \qquad (2\text{-}23)$$

which is the same as treated in Reference 24. A terminal wealth horizon posture constraint assumes that $G \geq k + g(w_1, \ldots, w_T)$, where k is a constant greater or equal than zero, and G is a function greater or equal than zero. Rewriting the equation thus,

$$-G + g\ (w_1, w_2, \ldots, w_T) \leq -k \qquad (2\text{-}24)$$

This does not allow the firm to have a negative terminal wealth. The model includes upper bounds on borrowed funds, the prohibition of multiple projects, and non-negativity of the variables. An analysis of the Kuhn-Tucker conditions yields interesting economic interpretations of the dual variables, similar to those presented in other models in the literature.

The Investment and Financing Decisions

Most of the models discussed in the academic literature assume a perfect capital market situation: there is no relation between investment project selection and the financing problem. This situation is either assumed or derived from axioms.[17] In reality, when a firm is borrowing funds the lender analyzes many aspects of the company's financial and managerial structure. Furthermore, restrictive covenants are often imposed after granting the loan, such as working-capital, fixed-asset, management, and dividend constraints. The question of whether or not the capital structure of the firm, represented by the mixture of long-term debt and equity used by the firm to finance its operations, affects its total valuation is directly related to the investment project decision making. The effect of changing the debt-equity ratio on the total valuation of the firm and to its cost of capital is explained by many authors on the basis of particular assumptions.[17]

The cost of capital of a corporation is the cost of obtaining funds, or, equivalently, the average return that an investor in a corporation expects after having invested proportionally in all the securities of the firm.[3] If it is assumed that the cost of capital is independent of the level of investment,[13] that is, it does not depend on a leverage measure (debt-equity ratio), the capital budgeting problem would be a simple task. The criterion for accepting or rejecting an investment project would require the comparison of the rate of return of the proposal with a fixed rate. However, because the cost of capital is a function of the level of investment, the corporation has to establish a capital structure and investment function.

Financial leverage may be defined as "the effect on the per share earnings of the common stock of a firm when large amounts must be paid for bond interest or preferred stock, or both, before the common stock is entitled to share in earnings."[3] A firm financed with stock only does not have leverage because all earnings are available for stockholders.

> It is generally agreed that as the proportion of debt increases and the capacity of the firm to service its debt without default decreases, the debt becomes a more risky investment in the mind of the lender, so that he will expect a higher interest rate to compensate for the increased risk.[10]

As pointed out by Haley and Schall in their book: The Theory of Financing Decision:

> There is a general agreement that equilibrium expected rates of return on debt and equity depend upon the proportion of debt financing used by the firm. As the proportion of debt increases, the probability of default may increase, bond prices may fall, and the expected rate

of return on the bonds may rise. Similarly, as the debt increases, the riskiness of the equity stream also rises and so will the associated required rate of return. Consequently, in a risk-averse market, the expected rates on both the debt and equity stream increase as the proportion of debt increases.

The above leads to the question of which method of financing to use. Corporations tend to use debt in addition to common stock mainly because the tax-deductible characteristics of the interest costs reduce the explicit cost of debt. From the viewpoint of capital structure it can be said that the firm that does not use long-term debt has an unused borrowing capacity. If the firm increases its long-term debt the interest rate paid will be constant over a range, and will rise after a certain point.[22] When bonds are issued, there is a basis for increasing the size and quantity of assets[3] which should cause higher levels of revenue and income to be available to the common shareholders, who have not increased in number. Beyond a certain point of leverage, the higher cost of debt and higher risk of default lead to greater equity return rates needed to maintain attractiveness of equity funds (Figure 2-2).

The corporation must make simultaneously the investment and financing decision to maximize the welfare of the stockholders, not separately. It is necessary to optimize investment project selection considering the required proportion of debt and equity financing, and a relationship relating a leverage rate to the borrowing rate should be included in the analysis.

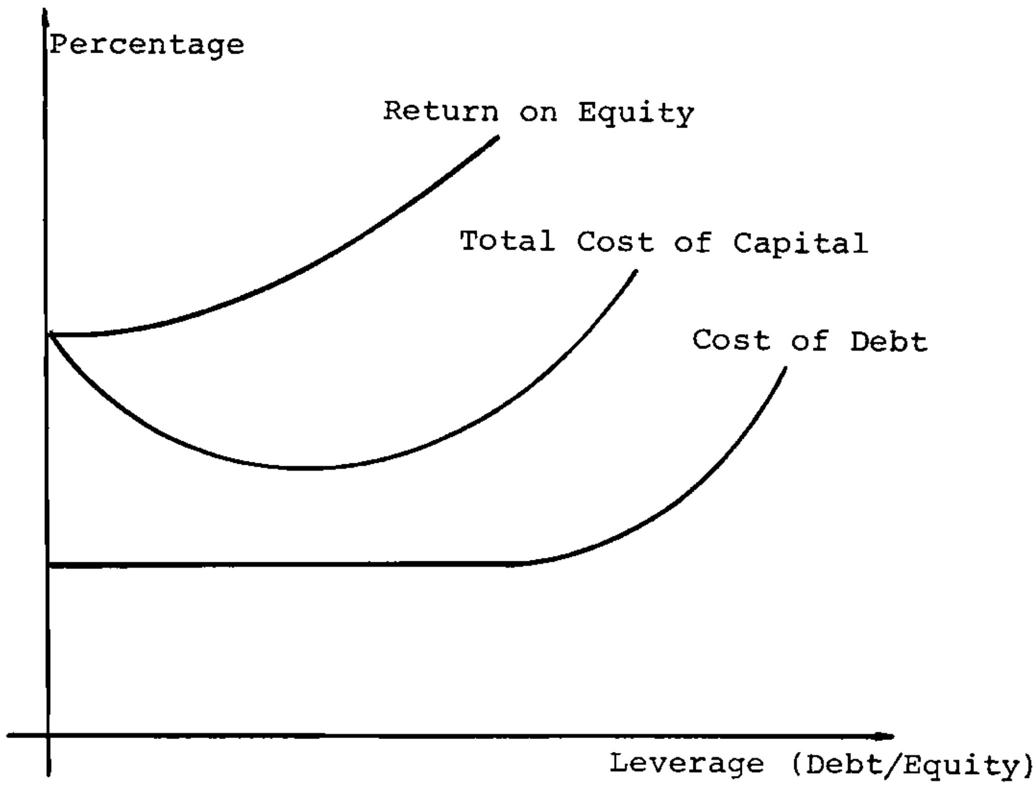


Figure 2-2. Leverage Effects.

Using the above concepts as a starting point, in the next chapter there is studied the relationship between the borrowing interest rate and the debt-equity ratio.

CHAPTER III

DATA COLLECTION AND REGRESSION ANALYSIS

In this chapter there is described the procedure followed in obtaining the relationship between the interest rate on borrowed money and the debt-equity ratio. Different options for expressing the debt-equity ratio are given, along with some relationships between the interest rate and other financial ratios.

Data Collection

In order to carry out the empirical study two important industries were chosen (Table 3-1):

1. Chemical industry
2. Multi-industry companies with chemical process operation.[20] The collected data correspond generally to operations during 1977, and were gathered directly from published annual reports. Tables 3-2 and 3-3 show this information for both industries.

The Debt-Equity Ratio

Three forms are considered for expressing the debt-equity ratio. The first one is the market value of debt over market value of equity. The market value of debt is the long-term debt of the firm computed from the current long-term yield of the firm's bonds traded in the New York Stock

Exchange, on the date of issue of the annual report for the firm. The market value of equity is the number of shares outstanding times the last price at which the stock was sold on the date of issue of the annual report.[8]

Expressing the ratio of market values reflects the fact that debt and equity shares are traded regularly. By this form the financial structure of the firm can be seen from the viewpoint of the market place.

The second alternative is to consider the book value of debt over book value of equity. Book value of debt is the redemption value of outstanding debt, and the interest rate is estimated as a weighted average of the long-term coupon rates associated with the debt. The book value of equity is considered to be the par value of common stock, capital surplus, and retained earnings, minus the carrying value of treasury stock and the liquidation value of preferred stock. This case reflects the viewpoint that the lender analyzes a prospective borrower by examining his financial statements. The debt values show how much the firm owes under different borrowing and repayment conditions. In comparing the market versus the book value of equity, it can be noted that in the former case the market value of common stock includes the value placed upon retained earnings.

The third alternative considered for expressing the debt-equity ratio is the ratio of book value of debt over market value of equity. Tables 3-4 and 3-5 show how the market value of equity is obtained and Tables 3-6 and 3-7 give the book and market value of equity, the current long-term yield[23] and the weighted average interest rate.

Borrowing Interest Rate-Debt/Equity Relationship

In order to find an empirical relationship between the debt-equity ratio and the borrowing interest rate, there can be suggested a function where the interest rate increases as the debt-equity ratio increases. Scatter diagrams suggest linear and logarithmic functions, as shown in Figure 3-1. The models can be defined as:

$$IR = a + b(w/E) + \varepsilon_t \qquad (3\text{-}1)$$

$$IR = a + c\ell n(w/E) + \varepsilon_t \qquad (3\text{-}2)$$

where IR = borrowing interest rate

a = intercept on IR axis

b = slope constant

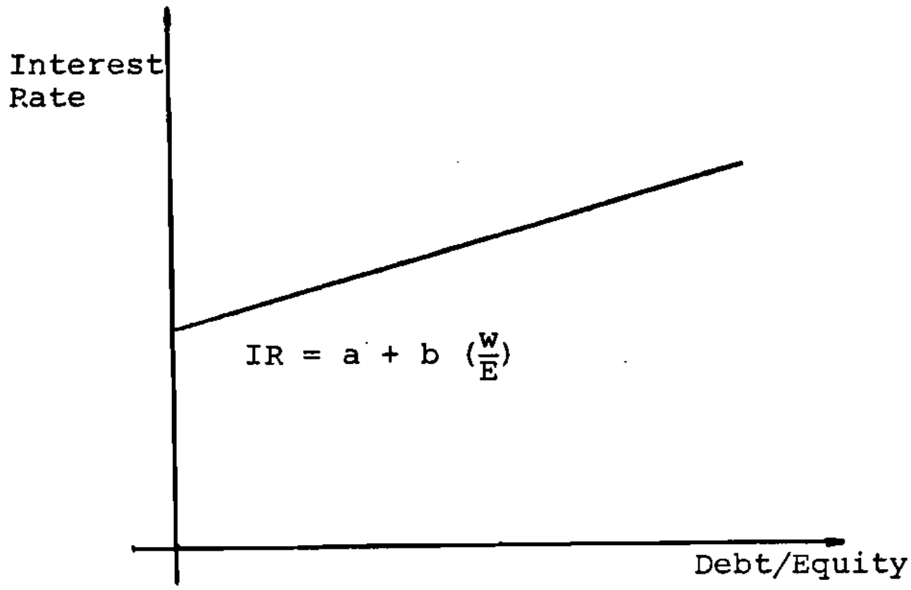
c = slope constant

w = long-term debt

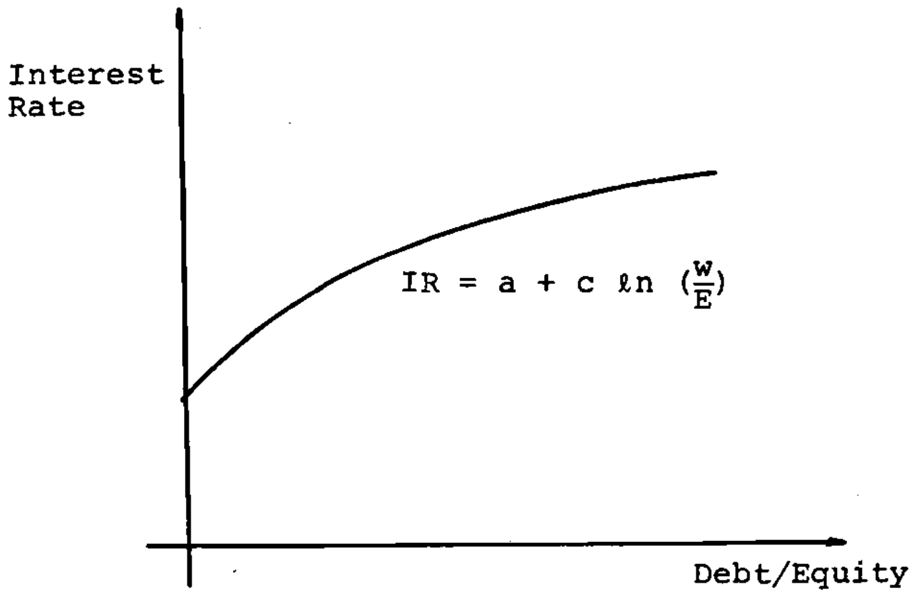
E = equity

ε_t = random deviation in period t.

It is assumed that ε_t is a normally distributed random variable with mean zero and unknown variance σ^2. Applying least squares methods, Tables 3-8 and 3-9 show the results obtained for both industries. The parameter b must be analyzed in order to demonstrate its statistical significance.



(3-1a)



(3-1b)

Figure 3-1. Linear and Logarithmic Relationships.

If negative, the parameter is assumed to be zero and is eliminated from the model. Considering the errors to be normally distributed, the overall significance of the regression model can be tested using the F statistic: (column 6, Tables 3-8 and 3-9)

$$F_0 = \frac{\sum_{j=1}^{n} (\hat{x}_j - \bar{x})^2/(k-1)}{\sigma_t^2} \tag{3-3}$$

where: $\bar{x}$ = the average of the x_j's

k = number of parameters in the model

x_j = the j^{th} data point

n = number of observations

σ_t^2 = estimator of the variance of the random error.

The value of F_0 must be compared with the upper α percentage point of the F distribution with (k-1) and (n-k) degrees of freedom, denoted by $F\alpha$, k-1, n-k (column 9, Tables 3-8 and 3-9). If $F_0 > F\alpha$, k-1, n-k, then at least one parameter in the model is significant. To test the significance of the parameter b_i, there is computed the statistic (column 5, Tables 3-8 and 3-9):

$$t_0 = \frac{\hat{b}_i}{\hat{\sigma}_t [\Sigma (x_i - \bar{x})^2]^{1/2}} \tag{3-4}$$

The $|t_0|$ is compared with $t\alpha/2$, n-k. If $|t_0| > t\alpha/2$, n-k, then the parameter b_i would be assumed to be nonzero. For this particular case, having one independent variable, the F-test is exactly the same as the t-test, such that $F = t^2$.[6]

Finally, the coefficient of determination R^2 is the proportion of total variation about the mean explained by the regression, usually expressed as a percentage.

Analysis of Result

Although the multi-industry group is more diversified in the types of business, the models obtained fit the relationship better than for the chemical industry models. It can also be observed that the process is better presented by a linear model for the multi-industry group and by a log-linear model for the chemical group.

After analyzing the F-test, there can be established the following set of relationships:

1. Chemical industry

For market value of debt over market value of equity

$$IR = 9.146 + 1.10 \ln (w/E) \qquad R^2 = 0.764 \qquad (3\text{-}5)$$

and for book value of debt over book value of equity:

$$IR = 8.655 + 1.38 \ln (w/E) \qquad R^2 = 0.265 \qquad (3\text{-}6)$$

2. Multi-industry group

For market value of debt over market value of equity

$$IR = 7.195 + 0.75 (w/E) \qquad R^2 = 0.417 \qquad (3\text{-}7)$$

for book value of debt over book value of equity

$$IR = 6.790 + 2.3595\ (w/E) \qquad R^2 = 0.628 \qquad (3\text{-}8)$$

and, for book value of debt over market value of equity

$$IR = 7.685 + 0.656\ (w/E) \qquad R^2 = 0.743 \qquad (3\text{-}9)$$

Figures 3-2 to 3-6 show how the model describes the process for each significant case. From these results, the relationship between a greater leverage and a higher borrowing interest rate is confirmed.[3,10,11]

Borrowing Interest Rate as a Function of Other Financial Ratios

In pursuing the main purpose of this chapter in finding an interest rate-debt/equity relationship, there were considered numerous other relationships involving various financial ratios.

Based on financial ratio analysis presented by Dun and Bradstreet,[7] and taking equity instead of tangible net-worth, Tables 3-11 and 3-13 show the set of financial ratios for the chemical industry considering book and market value of equity, respectively. Tables 3-10 and 3-12 do the same for the multi-industry group.

Stepwise multiple linear regression was used to find these relationships, with results as follows:

$$IR = 3.86 - 4.98\ \frac{\text{Net Profits}}{\text{Work. Capital}} + 7.93\ \frac{\text{Debt}}{\text{Equity}} + 0.45\ \frac{\text{Debt}}{\text{Equity}} \qquad R^2 = 0.7441 \qquad (3\text{-}10)$$

for market value of debt and equity:

$$IR = 7.52+3.41 \frac{\text{Net Profits}}{\text{Work. Capital}} - 2.35 \frac{\text{Current Liab.}}{\text{Equity}} \qquad (3\text{-}11)$$

$$R^2 = 0.8060$$

for the multi-industry group, considering book value for debt and equity:

$$IR = 11.81-0.74 \frac{\text{Curr. Assets-Inv.}}{\text{Curr. Liab.}} - 9.13 \frac{\text{Net Profit}}{\text{Equity}} - 2.83 \frac{\text{Curr. Liab.}}{\text{Equity}} \qquad R^2 = 0.9647 \qquad (3\text{-}12)$$

and for market value of debt and equity:

$$IR = 12.47-3.76 \frac{\text{Curr. Assets}}{\text{Curr. Liab.}} + 3.20 \frac{\text{Debt}}{\text{Equity}} - 2.15 \frac{\text{Fixed Assets}}{\text{Equity}} \qquad R^2 = 0.9045 \qquad (3\text{-}13)$$

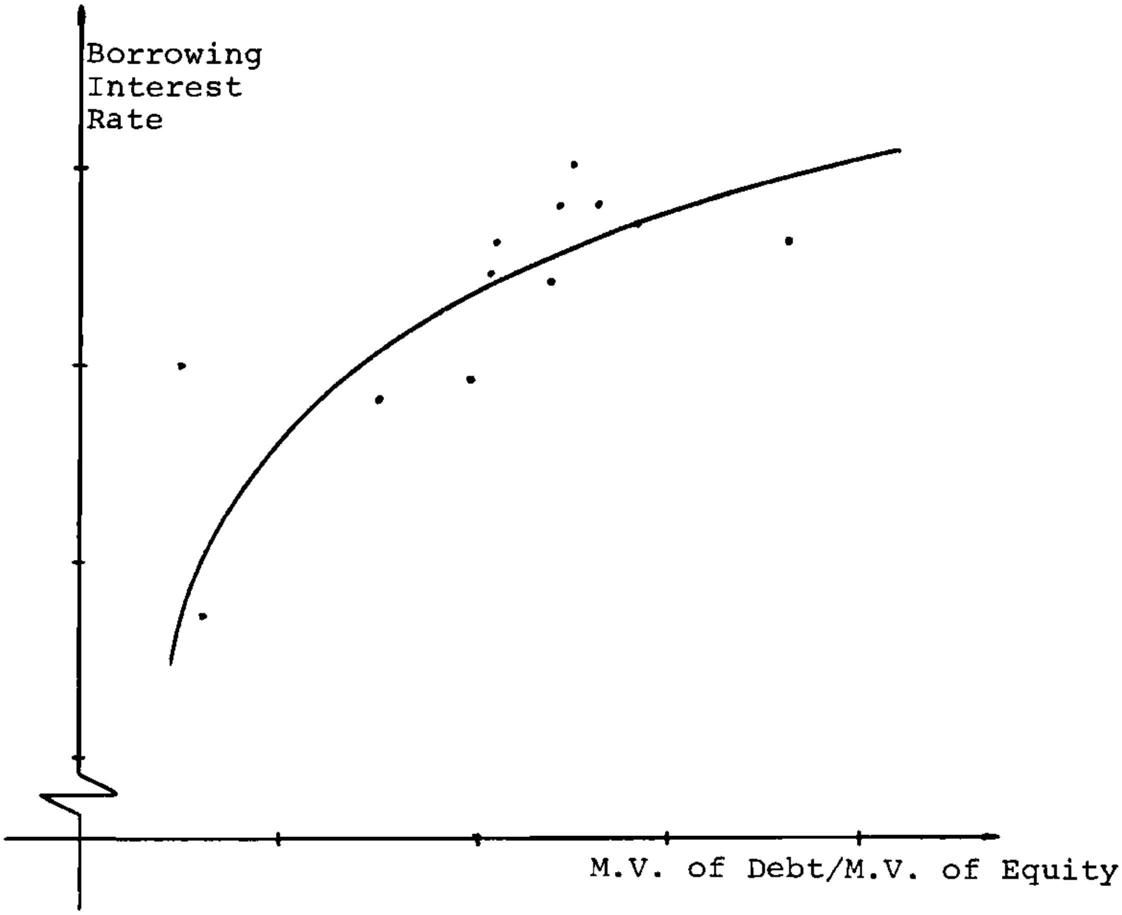


Figure 3-2. Chemical Group, Scatter Diagram.

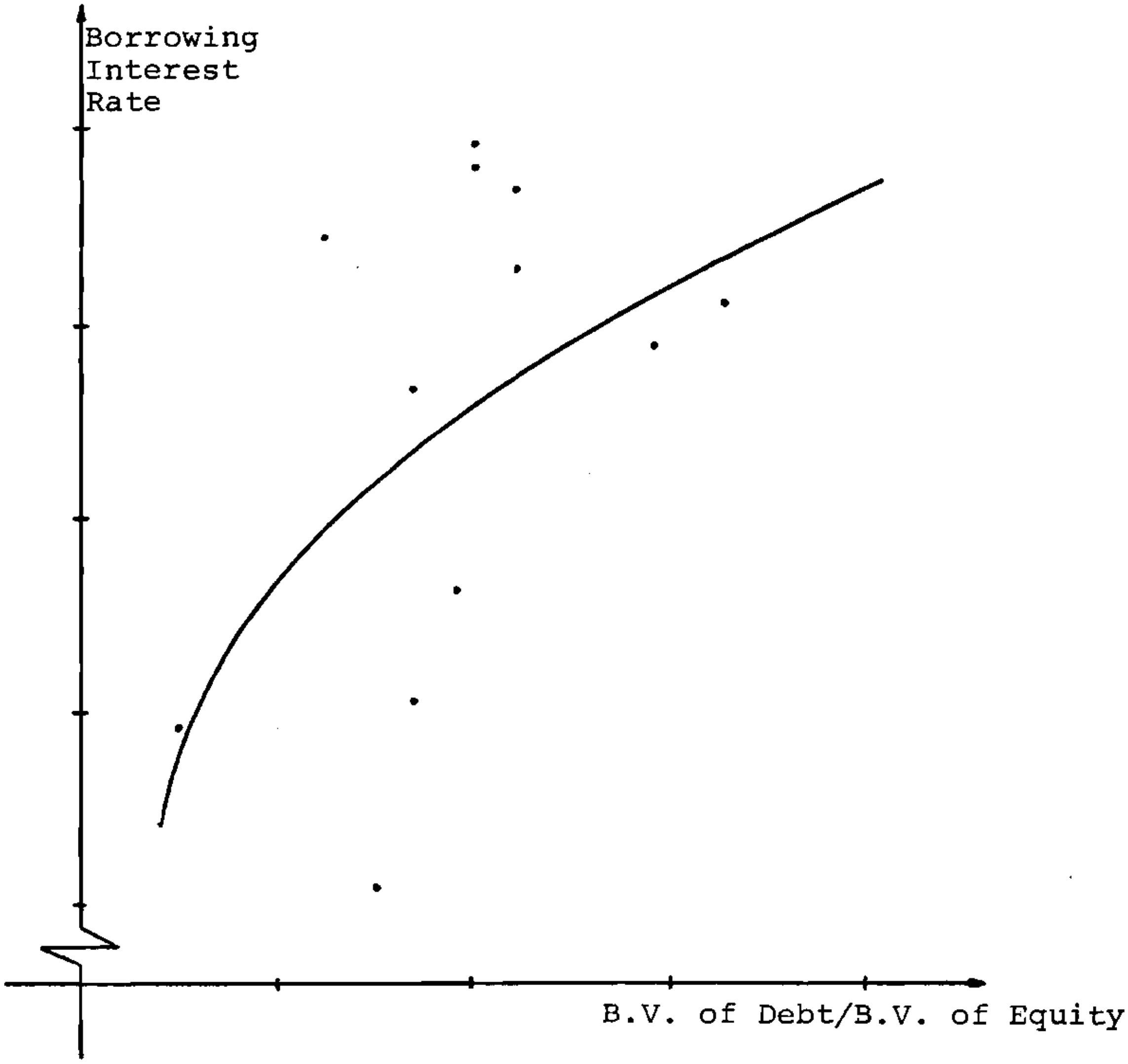


Figure 3-3. Chemical Group, Scatter Diagram.

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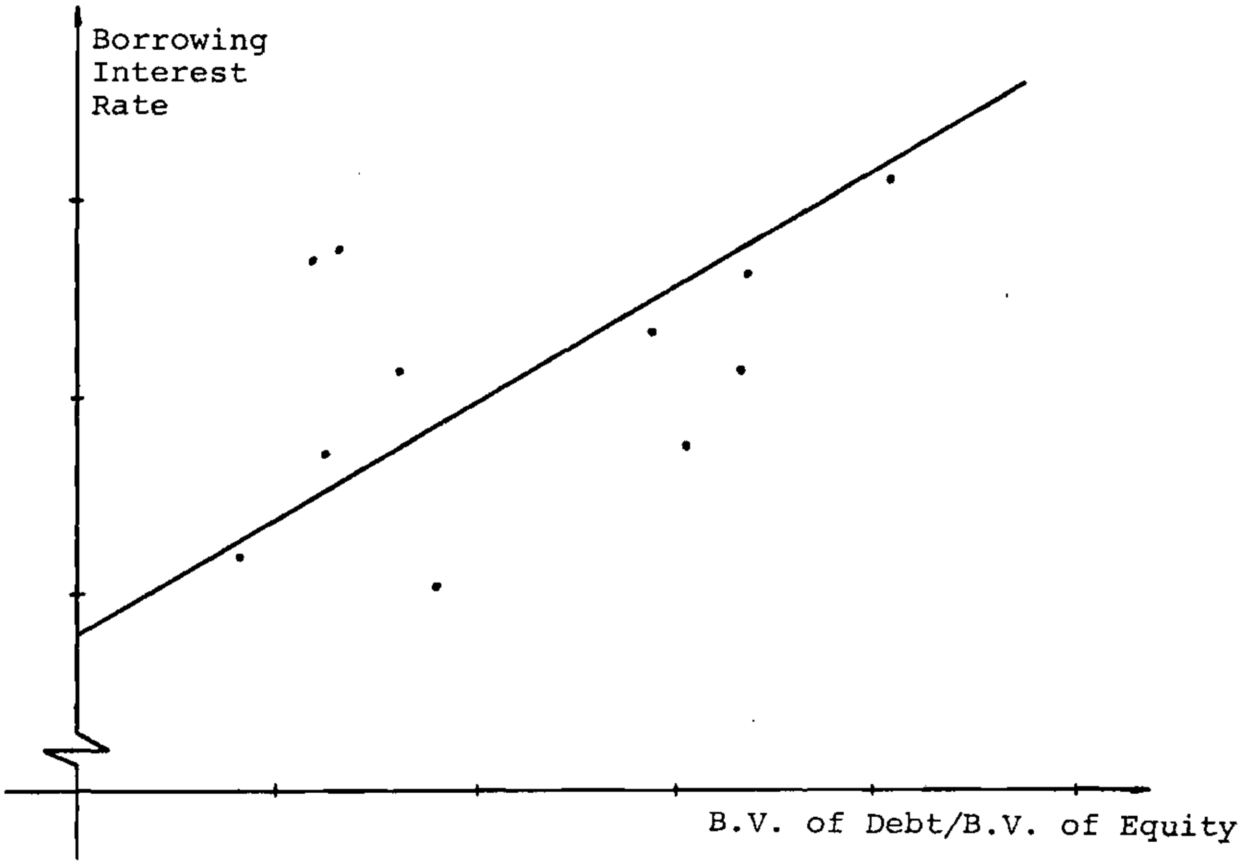

Figure 3-5. Multi-industry Group, Scatter Diagram.

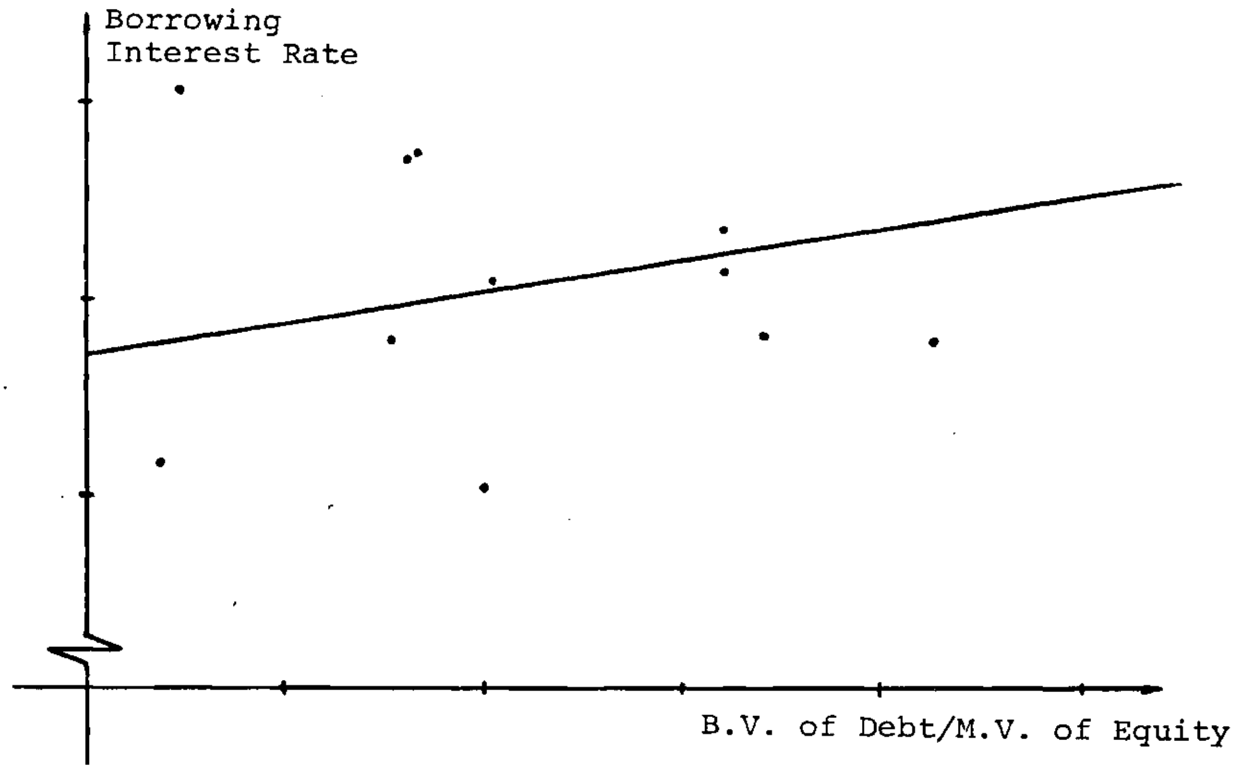

Figure 3-6. Multi-industry Group, Scatter Diagram.

Table 3-1. Companies Selected.

A. Multi-Industry Group

Number	Name
1	General Electric
2	International T. and T.
3	Tenneco Inc.
4	Textron Inc.
5	FMC Corp.
6	North American Philips
7	Northwest Industries
8	SCM Corp.
9	Wittacker Corp.
10	Sybron Corp.
11	Insilco Corp.
12	Texfi Industries
13	Martin Marietta
14	Carrier Corp.
15	National Distill. and Chemicals
16	Univar Corp.

B. Chemical Industry Group

Number	Name
1	Dow Chemical
2	Monsanto Co.
3	Ferro Corp.
4	American Cyanamid
5	Hercules Inc.
6	Union Carbide
7	Rohm and Haas
8	Diamond Shamrock
9	Dupont
10	Penwalt Corp.
11	Reichold Chemicals
12	Chemetron Corp.

Table 3-2. Chemical Industry Group, Financial Data.

	Current Assets	Inventories	Current Liabilities	Net Profits	Sales	Total Assets	Fixed Assets	Long-Term Debt	Equity
1	2,633,354	1,067,521	1,750,384	555,702	6,234,254	7,675,230	5,041,876	2,836,819	3,472,863
2	1,735,400	726,400	655,400	275,600	4,594,500	4,350,100	2,614,700	1,346,700	3,154,500
3	173,900	66,119	67,275	29,903	402,042	254,744	80,844	19,649	161,097
4	1,012,712	439,477	508,250	139,400	2,412,311	2,222,251	1,209,539	442,420	1,200,251
5	599,494	297,330	271,973	57,930	1,697,787	1,477,543	878,049	329,443	757,637
6	3,062,900	1,505,800	1,417,700	385,100	7,036,100	7,423,200	4,360,300	1,600,900	3,408,700
7	467,740	224,678	179,911	42,222	1,123,865	1,020,914	553,174	276,948	548,968
8	507,360	201,487	255,139	162,123	1,530,382	1,810,468	1,303,108	582,126	792,990
9	3,186,300	1,471,400	1,236,900	545,100	9,434,800	7,430,600	4,244,300	1,235,800	3,879,800
10	340,056	154,511	127,789	41,733	834,895	620,053	279,997	169,021	305,863
11	152,507	59,777	74,565	16,085	585,120	328,822	176,315	81,913	149,196
12	216,600	90,400	56,800	13,400	460,800	411,700	195,100	110,100	216,900

Table 3-3. Multi-industry Group, Financial Data.

	Current Assets	Inventories	Current Liabilities	Net Profits	Sales	Total Assets	Fixed Assets	Long-Term Debt	Equity
1	7,865,200	2,604,300	5,417,000	1,088,200	17,518,600	13,696,800	5,831,600	1,284,300	11,338,600
2	5,340,291	2,512,290	3,599,119	550,667	13,145,664	12,285,522	6,945,231	8,335,612	3,133,571
3	2,678,300	1,400,100	2,110,700	426,900	7,440,300	8,278,300	5,600,000	2,390,900	2,850,200
4	1,053,019	552,891	428,178	121,056	2,627,178	1,523,135	470,116	315,275	750,654
5	1,055,400	372,800	513,900	24,700	574,700	1,972,100	916,700	460,800	940,900
6	790,947	398,016	276,494	63,680	1,916,761	1,056,750	265,803	153,594	479,159
7	754,592	409,250	329,627	129,375	1,876,500	1,764,604	1,010,012	659,683	782,964
8	470,365	270,915	185,907	37,412	1,377,644	767,919	297,554	201,163	343,649
9	21,530	12,011	8,066	530	51,098	32,822	11,292	12,094	11,661
10	293,492	122,138	92,564	27,316	584,655	438,793	144,964	95,592	226,947
11	180,685	102,064	65,393	22,065	400,025	343,509	162,824	105,616	147,002
12	52,873	19,050	21,510	-12,941	182,875	127,095	74,222	66,635	36,981
13	604,972	209,194	321,489	102,110	1,439,761	1,376,778	771,806	218,873	763,853
14	605,633	252,660	296,039	57,142	1,310,430	914,039	308,406	173,150	386,636
15	761,005	353,451	263,793	84,998	1,587,167	1,293,318	523,313	237,519	709,430
16	178,070	84,400	100,674	-870	665,733	255,195	77,125	80,633	67,728

Table 3-4. Chemical Industry Group, Market Value of Equity.

	Firm	Shares Outstanding	Market Price	Mkt. Value of Equity
1	Dow Chemical	198,550,656	28	5,559,418,368
2	Monsanto Co.	36,872,323	56	2,064,850,088
3	Ferro Corp.	5,085,338	27	137,304,126
4	American Cyanamid	48,905,338	24	1,173,728,112
5	Hercules Inc.	42,386,717	16	678,187,472
6	Union Carbide	64,693,638	42	2,717,132,761
7	Rohm and Haas	13,108,730	30	393,261,900
8	Diamond Shamrock	39,966,384	28	1,119,058,752
9	Dupont	48,415,134	111	5,374,079,874
10	Penwalt Corp.	8,277,446	34	281,433,164
11	Reichold Chemicals	6,915,170	14	96,812,380
12	Chemetron Corp.	4,086,912	48	196,171,776

Table 3-5. Multi-industry Group, Market Value of Equity.

	Firm	Shares Outstanding	Market Price	Mkt. Value of Equity
1	General Electric	231,400,000	49	11,338,600,000
2	International T&T	104,452,384	30	3,133,571,520
3	Tenneco Inc.	92,956,464	30	2,788,693,920
4	Textron Inc.	29,937,828	25	748,445,700
5	FMC Corp.	31,935,266	24	765,461,328
6	North American Phil.	13,894,222	29	402,932,438
7	Norwest Industries	17,431,160	48	836,695,568
8	SCM Corp.	9,161,885	19	174,075,815
9	Whittacker Corp.	14,355,656	7	100,489,592
10	Sybron Corp.	10,539,031	17	179,163,527
11	Insilco Corp.	9,954,806	13	129,412,478
12	Texfi Industries	3,417,479	3	10,252,437
13	Martin Marietta	23,760,610	23	546,494,030
14	Carrier Corp.	24,207,000	14	338,898,000
15	National Distill. & Chem.	25,899,080	22	569,779,760
16	Univar Corp.	6,698,844	9	60,289,596

Table 3-6. Multi-industry Group, Debt-Equity Ratios.

Firm	Equity x 10^3 (Book Value)	Equity x 10^3 (Mkt. Value)	Long-Term Debt x 10^3	L-T Debt / B.V. of Equity	L-T Debt / M.V. of Equity	Current Yield %	Weighted Ave Int. Rate %
General Electric	6,108,400	11,338,600	1,284,300	0.21	0.11	7.50	7.19
International T&T	4,363,472	3,133,572	3,335,612	0.76	1.06	8.40	7.66
Tenneco Inc.	2,850,200	2,788,694	2,390,900	0.84	0.85	8.60	7.76
Textron Inc.	750,000	748,446	315,275	0.42	0.44	8.50	-
FMC Corp.	940,000	765,461	460,800	0.49	0.60	6.20	-
North American Philips	479,159	402,932	153,594	0.32	0.38	5.70	7.72
Norwest Industries	782,964	836,676	659,683	0.84	0.78	8.00	8.09
SCM Corp.	343,649	174,076	201,163	0.59	1.15	8.10	-
Whittacker Corp.	11,661	100,490	12,090	1.03	0.12	10.00	9.03
Sybron Corp.	226,947	179,164	92,592	0.41	0.51	5.80	8.07
Insilico Corp.	147,002	129,412	105,616	0.72	0.80	8.60	8.37
Texfi Industries	36,981	10,252	66,633	1.80	6.49	11.87	12.10
Martin Marietta	763,636	546,494	218,873	0.29	0.40	6.50	8.66
Carrier Corp.	386,636	338,898	173,150	0.45	0.50	8.40	6.98
National Distillers	709,430	569,780	237,519	0.33	0.41	5.30	8.74
Univar Corp.	67,728	60,290	80,633	1.19	1.33	9.60	-

Table 3-7. Chemical Industry Group, Debt-Equity Ratios.

Firm	Equity (Book Value)	Equity (Mkt. Value)	Long-Term Debt x 10^3	L-T Debt / B.V. of Equity	L-T Debt / M.V. of Equity	Current Yield %	Weighted Ave Int. Rate %
Dow Chemical	3,472,863	5,559,418	2,836,819	0.82	0.51	8.45	8.09
Monsanto Co.	3,154,500	2,064,850	1,346,700	0.43	0.65	8.80	6.05
Ferro Corp.	161,097	137,304	19,649	0.12	0.14	6.70	5.88
American Cyanamid	1,200,000	1,173,728	442,420	0.37	0.37	7.80	5.07
Hercules Inc.	757,637	678,187	329,443	0.43	0.48	7.90	7.62
Union Carbide	3,408,700	2,717,133	1,600,000	0.47	0.58	8.40	6.59
Rohm and Haas	548,958	393,262	276,948	0.50	0.70	8.70	8.81
Diamond Shamrock	792,990	1,119,059	582,126	0.73	0.52	8.60	7.91
Dupont	3,879,800	5,374,080	1,235,800	0.32	0.23	8.00	8.43
Pennault Corp.	305,863	281,433	169,021	0.55	0.60	8.80	8.27
Reichold Chemicals	149,196	96,812	81,913	0.55	0.84	8.60	8.72
Chemetron Corp.	216,900	196,171	110,100	0.51	0.56	9.00	8.94

Table 3-8. Multi-industry Group, Statistical Results.

Option	Equation	R^2	Std. Error of "b"	t_o	F_0	d_{fn}	d_{fd}	F_c
1	IR = 7.195 + 0.750 (w/E)	0.417	0.237	3.164	10.01	1	14	4.60
1	IR = 8.378 + 0.863 ℓn(w/E)	0.218	0.437	1.976	3.90	1	14	4.60
2	IR = 6.790 + 2.3595 (w/E)	0.628	0.573	4.111	16.90	1	10	4.96
2	IR = 9.138 + 1.312 ℓn(w/E)	0.387	0.522	2.513	6.32	1	10	4.96
3	IR = 7.685 + 0.656 (w/E)	0.743	0.122	5.373	28.88	1	10	4.96
3	IR = 8.827 + 0.775 ℓn(w/E)	0.379	0.314	2.469	6.09	1	10	4.96

IR = Interest rate
w = Debt
E = Equity
d_{fn} = Degrees of freedom of numerator
d_{fd} = Degrees of freedom of denominator
F_c = Critical value of F-test

Options

1 - M.V. of debt/M.V. of equity
2 - B.V. of debt/B.V. of equity
3 - B.V. of debt/M.V. of equity

Table 3-9. Chemical Industry Group, Statistical Results.

Option	Equation	R^2	Std. Error of "b"	t_0	F_0	d_{fn}	d_{fd}	F_c
1	IR = 6.959 + 2.628 (w/E)	0.649	0.611	4.301	18.51	1	10	4.96
1	IR = 9.146 + 1.101 ℓn(w/E)	0.764	0.194	5.685	32.32	1	10	4.96
2	IR = 5.810 + 3.563 (w/E)	0.245	1.977	1.802	3.25	1	10	4.96
2	IR = 8.656 + 1.380 ℓn(w/E)	0.265	0.726	1.900	3.61	1	10	4.96
3	IR = 8.342 + 1.069 (w/E)	0.170	0.746	1.432	2.05	1	10	4.96
3	IR = 6.042 + 2.896 ℓn(w/E)	0.186	1.914	1.513	2.29	1	10	4.96

IR = Interest rate
w = Debt
E = Equity
d_{fn} = Degrees of freedom of numerator
d_{fd} = Degrees of freedom of denominator
F_c = Critical value of F-test

Options

1 - M.V. of debt/M.V. of equity
2 - B.V. of debt/B.V. of equity
3 - B.V. of debt/M.V. of equity

Table 3-10. Multi-industry Group, Financial Ratios Considering Book Value of Debt and Equity.

	CA / CL	CA-Inv / CL	Net Profits / Work. Capital	Sales / Work. Capital	Sales / Inv	L-T Debt / Equity	Net Profits / Equity	Sales / Equity	Fixed Assets / Equity	CL / Equity
1	1.45	0.97	0.44	7.16	6.73	0.11	0.10	1.55	0.51	0.48
2	1.48	0.78	0.32	7.55	5.23	1.06	0.18	4.20	2.22	1.15
3	1.27	0.60	0.75	3.00	1.22	0.85	0.15	0.61	2.01	0.76
4	2.45	1.17	0.19	4.20	4.75	0.44	0.16	3.51	0.63	0.57
5	2.05	1.32	0.05	1.06	1.54	0.60	0.03	0.75	1.20	0.67
6	2.86	1.42	0.16	4.88	4.82	0.38	0.16	4.76	0.66	0.69
7	2.29	1.04	0.30	4.42	4.59	0.78	0.15	2.24	1.21	0.39
8	2.53	1.07	0.13	4.84	5.09	1.15	0.21	7.91	1.71	1.07
9	2.67	1.18	0.04	3.80	4.25	0.12	0.01	0.51	0.11	0.08
10	3.17	1.85	0.14	2.91	4.79	0.51	0.15	3.26	0.81	0.52
11	2.76	1.20	0.28	5.09	3.92	0.80	0.17	3.09	1.26	0.51
12	2.45	1.57	-0.41	5.83	9.60	6.49	-1.26	17.84	7.24	2.10
13	1.88	1.23	0.36	5.08	6.88	0.40	0.19	2.63	1.41	0.59
14	2.05	1.19	0.18	4.23	5.19	0.50	0.17	3.87	0.91	0.87
15	2.88	1.54	0.17	3.19	4.49	0.41	0.15	2.79	0.93	0.46
16	1.77	0.93	-0.01	8.60	7.88	1.33	-0.01	11.04	1.28	1.67

CA = Current Assets
CL = Current Liabilities
Inv = Inventories

Table 3-11. Chemical Industry Group, Financial Ratios Considering Book Value of Debt and Equity.

	CA / CL	CA-Inv / CL	Net Profits / Work. Capital	Sales / Work. Capital	Sales / Inv.	L-T Debt / Equity	Net Profits / Equity	Sales / Equity	Fixed Assets / Equity	CL / Equity
1	1.50	0.84	0.63	7.06	5.84	0.51	0.10	1.12	0.91	0.31
2	2.64	1.54	0.26	4.25	6.33	0.65	0.13	2.23	1.27	0.32
3	2.58	1.60	0.20	3.94	6.35	0.14	0.15	3.06	0.59	0.49
4	1.99	1.13	0.28	4.78	5.49	0.37	0.12	2.06	1.03	0.43
5	2.20	1.11	0.18	5.18	5.71	0.48	0.09	2.50	1.29	0.40
6	2.16	1.10	0.23	4.28	4.67	0.58	0.14	2.59	1.60	0.52
7	2.59	1.35	0.15	3.90	5.00	0.70	0.11	2.86	1.41	0.46
8	1.98	1.20	0.64	6.07	7.60	0.52	0.14	1.37	1.16	0.23
9	2.57	1.39	0.28	4.84	6.41	0.23	0.10	1.76	0.79	0.23
10	2.66	1.45	0.20	3.93	5.40	0.60	0.15	2.97	0.99	0.45
11	2.04	1.24	0.21	3.92	9.79	0.84	0.17	6.04	1.82	0.77
12	3.81	2.22	0.08	2.88	5.10	0.56	0.17	2.35	0.99	0.29

CA = Current Assets
CL = Current Liabilities
Inv = Inventories

Table 3-12. Multi-industry Group, Financial Ratios Considering Market Value of Debt and Equity.

	CA / CL	CA-Inv / CL	Net Profits / Work. Capital	Sales / Work. Capital	Sales / Inv	L-T Debt / Equity	Net Profits / Equity	Sales / Equity	Fixed Assets / Equity	CL / Equity
1	1.45	0.97	0.44	7.16	6.73	0.21	0.18	2.87	0.95	1.06
2	1.48	0.78	0.32	7.55	5.23	0.76	0.13	3.01	1.59	1.44
3	1.27	0.60	0.75	3.00	1.22	0.84	0.15	0.60	1.96	2.47
4	2.45	1.17	0.19	4.20	4.75	0.42	0.16	3.50	0.63	0.88
5	2.05	1.32	0.05	1.06	1.54	0.49	0.03	0.61	0.97	0.69
6	2.85	1.42	0.16	4.88	4.82	0.32	0.13	4.00	0.55	1.01
7	2.29	1.04	0.30	4.42	4.59	0.84	0.17	2.40	1.29	0.96
8	2.53	1.07	0.13	4.84	5.09	0.59	0.11	4.01	0.87	0.95
9	2.67	1.18	0.04	3.80	4.25	1.03	0.05	4.38	0.97	0.89
10	3.17	1.85	0.14	2.91	4.79	0.41	0.12	2.58	0.64	0.61
11	2.76	1.20	0.28	5.09	3.92	0.72	0.15	2.72	1.11	1.30
12	2.45	1.57	-0.41	5.83	9.60	1.80	-0.35	4.95	2.01	0.61
13	1.88	1.23	0.36	5.08	6.88	0.29	0.13	1.88	1.01	0.74
14	2.05	1.19	0.18	4.23	5.19	0.45	0.15	3.39	0.80	0.82
15	2.88	1.54	0.17	3.19	4.49	0.33	0.12	2.26	0.75	0.71
16	1.77	0.93	-0.01	8.60	7.88	1.19	-0.01	9.83	1.14	1.09

CA = Current Assets
CL = Current Liabilities
Inv = Inventories

Table 3-13. Chemical Industry Group, Financial Ratios Considering Market Value of Debt and Equity.

	CA/CL	CA-Inv/CL	Net Profits/Work. Capital	Sales/Work. Capital	Sales/Inv	L-T Debt/Equity	Net Profits/Equity	Sales/Equity	Fixed Assets/Equity	CL/Equity
1	1.50	0.89	0.63	7.06	5.84	0.82	0.16	1.80	1.45	0.50
2	2.64	1.54	0.26	4.25	6.33	0.43	0.09	1.46	0.83	0.21
3	2.58	1.60	0.20	3.94	6.35	0.12	0.13	2.61	0.50	0.42
4	1.99	1.13	0.28	4.78	5.49	0.37	0.11	2.01	1.01	0.42
5	2.20	1.11	0.18	5.18	5.71	0.43	0.08	1.95	1.16	0.36
6	2.16	1.10	0.23	4.28	4.67	0.47	0.11	2.18	1.28	0.42
7	2.59	1.35	0.15	3.90	5.00	0.50	0.08	2.05	1.01	0.33
8	1.98	1.20	0.64	6.07	7.60	0.73	0.20	1.93	1.64	0.32
9	2.57	1.39	0.28	4.84	6.41	0.32	0.14	7.63	0.34	1.00
10	2.66	1.45	0.20	3.93	5.40	0.55	0.14	2.73	0.92	0.42
11	2.04	1.24	0.21	3.92	9.79	0.55	0.11	3.92	1.18	0.50
12	3.81	2.22	0.08	2.88	5.10	0.51	0.06	2.12	0.90	0.26

CA = Current Assets
CL = Current Liabilities
Inv = Inventories

CHAPTER IV

MODEL FORMULATION AND OPTIMALITY ANALYSIS

In this chapter there are presented three models for deterministic capital budgeting with inclusion of some type of relationship between borrowing interest rate and debt/ equity ratio. The relationships used are based on the regression results obtained in Chapter III. The definition of equity for each period is described below.

The first and most general model is a non-convex program. A simplification results in the second model, a convex program, and further simplification gives the third model, a linear program (LP). An analysis is made of the optimality conditions for each model, with a view toward determining when global optima are obtained and for economic interpretations.

Notation

Let:

a_{tj} = Net cash flow obtainable from a unit of project j at time t, $(t = 0, 1, \ldots, T)$, $(j = 1, 2, \ldots, n)$. Revenues are positive flows and expenditures are negative flows.

M_t = Amount of cash available for investment from outside sources at time t, $(t = 0, 1, \ldots, T)$.

r_ℓ = Lending interest rate, greater than zero.

r_{bt} = Borrowing interest rate from time t to time t + 1, greater than zero, (t = 0, 1,..., T).

w_t = Cash to be borrowed from time t to t + 1, (t = 0, 1,... T).

v_t = Cash to be lent from time t to t + 1, (t = 0, 1,..., T).

x_j = Number of units of project j to be undertaken.

An asterisk implies the optimal value of a variable.

General Model: Nonlinear Programming Problem

Weingartner's basic horizon model is taken as a point of departure in the formulation of the general model. Next are discussed the objective function and the constraints involved in the formulation.

Objective Function

The objective function to be maximized is the time T terminal wealth of the firm, considered as the net value of assets at time T, which are expressed as the funds available for lending at that time plus the value of any post-horizon cash flows expected from projects. Thus, the objective function is:

$$G = v_T - w_T \tag{4-1}$$

The w_T term presents a cash deficit position at time T.

Constraints

Five types of constraints define the feasible solution space. The dual variable associated with each constraint is shown in brackets at the left side of the equation.

1. Cash Balance Constraints. The cash balance constraint specifies that the net cash outflow of the projects plus the cash outflow for time t loans, minus the cash inflow for time t borrowing, minus the cash inflow for repayment of time t-1 loans, plus the cash outflow for repayment of time t-1 borrowing, must be less than or equal to the cash available from outside sources at time t. Then, the form of the cash balance constraint is:

$$[u_{Mt}] \qquad - \sum_{j=1}^{n} a_{tj}x_j + v_t - w_t - v_{t-1}(1+r\ ) + w_{t-1}(1+r_{bt-1}) \leq M_t \qquad t = 1,2,\ldots, T. \qquad (4\text{-}2)$$

for time zero, the expression is:

$$- \sum_{j=1}^{n} a_{0j}x_j + v_0 - w_0 \leq M_0 \qquad (4\text{-}3)$$

2. Equity Constraints. In defining the equity constraints there is assumed an initial fixed equity denoted by E_0. If the initial equity were a variable, the problem would represent an unrealistic situation, because the equity variables would tend to take extremely high values in order to reduce the debt-equity ratio and make the borrowing interest rate very low. This situation was demonstrated computationally. The equity at time t is defined to be equal to the

equity at time t-1, plus the interest earned on lending activities from time t to t+1, minus the interest paid on borrowing activities from time t to t+1, plus the cash from projects at time t.

In this way it is assumed that the equity of the firm during the horizon changes only due to the cash available from projects and interest on borrowing and lending. Tax effects are not explicitly considered, since the a_{tj}, r_{ℓ}, and r_{bt} can be defined for after-tax situations. Dividends and the issuance and purchase of stock are not considered. The above formulation can be related to the beginning of a new firm, which starts with an initial equity and an investment project set. The equity constraint is expressed as an inequality so that optimality analysis may be more convenient. This change does not affect the solution since any algorithm will tend to set equity values as high as possible, resulting in equality of the constraint, to keep borrowing interest rates low. Thus, the equity constraint can be expressed as:

$$E_t \leq E_{t-1} + v_{t-1}r_{\ell} - w_{t-1}r_{bt-1} + \sum_{j=1}^{n} a_{tj}x_j \qquad (4\text{-}4)$$

$$t = 0, 1, 2, \ldots, T.$$

Rearranging terms:

$$[u_{Et}] \quad E_t - E_{t-1} - v_{t-1}r_\ell + w_{t-1}r_{bt-1} - \sum_{j=1}^{n} a_{tj}x_j \leq 0 \qquad (4\text{-}5)$$

$$t = 0,1,\ldots T.$$

3. Interest Rate Constraints. This restriction simply specifies the interest rate at which the firm can borrow money. The linear relationship developed in Chapter III is used here. Again, the equality is relaxed to an inequality:

$$r_{bt} \geq a + b\left(\frac{w_t}{E_t}\right) \qquad (4\text{-}6)$$

Rearranging terms:

$$[u_{it}] \qquad bw_t + E_t(a - r_{bt}) \leq 0 \qquad t = 0, 1, \ldots, T. \qquad (4\text{-}7)$$

4. Project Upper Bounds. These constraints eliminate the possibility of undertaking more than one of a given project.

$$[u_{xj}] \qquad x_j \leq 1 \qquad j = 1,2,\ldots, n. \qquad (4\text{-}8)$$

5. Non-Negativity Constraints. It is not permitted to have negative values of the variables, which would represent an unrealistic situation. Thus:

$$v_t,\ w_t,\ E_t,\ r_{bt} \geq 0 \qquad t = 0,1,\ldots, T \qquad (4\text{-}9)$$

$$x_j \geq 0 \qquad j = 1, 2,\ldots, n \qquad (4\text{-}10)$$

Summarizing, the general model is expressed as:

$$\text{maximize:} \quad v_T - w_T \tag{4-1}$$

$$\text{subject to:} \quad -\sum_{j=1}^{n} a_{0j}x_j + v_0 - w_0 \leq M_0 \tag{4-3}$$

$$-\sum_{j=1}^{n} a_{tj}x_j + v_t - w_t - v_{t-1}(1+r_\ell) + w_{t-1}(1+r_{bt-1}) \leq M_t \qquad t = 1,2,\ldots, T \tag{4-2}$$

$$E_1 - \sum_{j=1}^{n} a_{1j}x_j - v_0r_\ell + w_0r_{b0} \leq E_0 \tag{4-11}$$

$$E_t - E_{t-1} - \sum_{j=1}^{n} a_{tj}x_j - v_{t-1}r_\ell + w_{t-1}r_{bt-1} \leq 0 \qquad t = 1,2,\ldots, T \tag{4-5}$$

$$E_t(a-r_{bt}) + bw_t \leq 0 \qquad t = 0,1,\ldots, T \tag{4-7}$$

$$0 \leq x_j \leq 1 \qquad j = 1,2,\ldots, n \tag{4-12}$$

$$v_t,\ w_t,\ r_{bt} \geq 0 \qquad t = 0,1,\ldots, T \tag{4-10}$$

The above is a nonlinear programming problem with a linear objective function and four types of significant constraints, three of them nonlinear.

<u>Optimality Analysis for the Nonlinear Programming Problem</u>

For a nonlinear programming problem of the form:[12]

$$\text{maximize:} \quad f(x) \tag{4-13}$$

$$\text{subject to:} \quad g_i(x) \le b_i \qquad i = 1,2,\ldots, m \tag{4-14}$$

$$x_j \ge 0 \qquad j = 1,2,\ldots, n \tag{4-15}$$

where $f(x)$ and $g_i(x)$ are differentiable functions, $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is a Kuhn-Tucker point if there exist $u_1, u_2, \ldots, u_m$ such that the following conditions are satisifed.

$$\frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{m} \frac{\partial g_i(x)}{\partial x_j} \le 0 \tag{4-16}$$

$$\text{at } x_j = x_j^*, \ldots, j=1,2,\ldots,n$$

$$x_j^*\left(\frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{m} \frac{\partial g_i(x)}{\partial x_j}\right) = 0 \tag{4-17}$$

$$g_i(x^*) - b_i \le 0 \tag{4-18}$$

$$i = 1,2,\ldots, m$$

$$u_i(g_i(x^*) - b_i) = 0 \tag{4-19}$$

$$x_j^* \ge 0 \qquad j = 1,2,\ldots, n \tag{4-20}$$

$$u_i \ge 0 \qquad i = 1,2,\ldots, m \tag{4-21}$$

These conditions are called the Kuhn-Tucker conditions, where the u_i's are referred to as Lagrange Multipliers. The above expressions are necessary conditions for optimality, but not sufficient. The problem must satisfy certain convexity assumptions to guarantee a global optimal point: if $f(x)$ is a concave function and $g_1(x)$, $g_2(x)$,... $g_m(x)$ are convex functions, then $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ is a global optimal solution.

Checking the Sufficient Conditions for Optimality

In order to ensure that the solution obtained by solving the nonlinear programming problem is a global optimal solution, it is necessary to analyze the objective function, which must be concave, and the set of constraints, which must form a convex set. Examining Equation (4-1), it is seen to be a linear function, and hence, a concave function.

For analyzing the set of constraints there is performed a test of convexity: a function $g(x)$ is convex if the Hessian of $g(x)$ is positive definite or positive semidefinite for all non-zero vectors $\underline{x} = (x_1, x_2, \ldots, x_n)$. Before performing the analysis it is necessary to define some terms used in the convexity analysis:[18]

a. The Hessian of a function $g(x_1, x_2, \ldots, x_n)$ is a $(n \times n)$ symmetric matrix given by:

$$H_g(x_1, x_2, \ldots, x_n) = [\frac{\partial^2 g(x)}{\partial x_i \partial x_j}] \tag{4-22}$$

b. A matrix H is positive definite if and only if the quadratic form:

$$\underline{x}\ H\ \underline{x}' > 0 \text{ for all } \underline{x} \neq 0 \qquad (4\text{-}23)$$

where $\underline{x}'$ is the transpose of vector $\underline{x}$. It is positive semidefinite if and only if $\underline{x}\ H\ \underline{x}' \geq 0$ for all $\underline{x} \neq 0$ and there exists an $\underline{x} \neq 0$ such that $\underline{x}\ H\ \underline{x}' = 0$.

c. A matrix is negative definite or negative semidefinite if and only if -H is positive definite or positive-semidefinite, respectively.

Analysis of Budget Constraints. The general expression for the budget constraint is:

$$-\sum_{j=1}^{n} a_{tj}x_j + v_t - w_t - v_{t-1}(1+r_\ell) + w_{t-1}(1+r_{bt-1}) \leq M_t \qquad (4\text{-}2)$$

$$t = 1,2,\ldots, T$$

The Hessian for this function, denoted by H_{BC}, is

$$H_{BC} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \\ v_{t-1} \\ v_t \\ w_{t-1} \\ w_t \\ r_{bt-1} \end{array} \begin{array}{c} \begin{array}{ccccccccc} x_1 & x_2 & \ldots & x_n & v_{t-1} & v_t & w_{t-1} & w_t & r_{bt-1} \end{array} \\ \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad (4\text{-}24)$$

Applying Equation (4-23) where $x = (x_1, x_2, \ldots, x_n, v_{t-1}, v_t, w_{t-1}, w_t, r_{bt-1})$, there is obtained the expression $2r_{bt-1}\, w_{t-1}$ which is greater than zero for any r_{bt-1} and w_{t-1} greater than zero. Thus H_{BC} is positive definite and this constraint defines a convex set.

Analysis of Equity Constraints. The equity constraints are given by the following expression:

$$E_t - E_{t-1} - v_{t-1}r_\ell + w_{t-1}r_{bt-1} - \sum_{j=1}^{n} a_{tj}x_j \le 0 \qquad t = 1,2,\ldots, T \tag{4-5}$$

The Hessian H_{EC} can be stated as:

$$H_{EC} = \begin{array}{c|cccccccccc} & x_1 & x_2 & \cdots & x_n & E_{t-1} & E_t & v_{t-1} & w_{t-1} & r_{bt-1} \\ \hline x_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ x_2 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{t-1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ E_t & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ v_{t-1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{t-1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & 1 \\ r_{bt-1} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \tag{4-25}$$

Again applying Equation (4-23), where $x = (x_1, x_2, \ldots, x_n, E_{t-1}, E_t, v_{t-1}, w_{t-1}, r_{bt-1})$ the following expression is obtained:

$$2r_{bt-1}w_{t-1} > 0 \text{ for all } r_{bt-1}, w_{t-1} > 0 \qquad (4\text{-}26)$$

So H_{EC} is positive definite and this constraint also defines a convex set.

Analysis of Interest Rate Constraints. The general form of this constraint is:

$$bw_t + E_t(a - r_{bt}) \leq 0 \qquad (4\text{-}7)$$

The Hessian H_{IRC} can be stated as:

$$H_{IRC} = \begin{array}{c c} & \begin{array}{ccc} w_t & E_t & r_{bt} \end{array} \\ \begin{array}{c} w_t \\ E_t \\ r_{bt} \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} \end{array} \qquad (4\text{-}27)$$

Applying Equation (4-23) one obtains the following expression where $x = (w_t, E_t, r_{bt})$

$$-2r_{bt}E_t < 0 \text{ for all } r_{bt}, E_t > 0 \qquad (4\text{-}28)$$

Thus, H_{IRC} is negative definite and this set of constraints does not define a convex set.

The constraints of the type $x_j \leq 1$ for $j = 1,2,\ldots, n$ are linear functions and so they can be considered as convex functions.

After analyzing the set of constraints, it can be said that they define a non-convex set. Therefore, the sufficient conditions for optimality cannot be used to assure that an optimal solution corresponds to a global optimal point. Any algorithm employed in obtaining the solution of this non-convex nonlinear programming problem will reach a point that might be a local optimal point. In the next chapter the problem is solved with different starting points in an attempt to investigate the possibility of obtaining the same solution, and if not, then to increase the probability of obtaining the optimal solution.

Modification of the General Problem

Some changes can be suggested in the formulation of the problem in order to transform the non-convex into a convex nonlinear programming problem. If it is assumed that the equity of the firm remains constant over the horizon, the equity constraints can be deleted and the problem stated as:

$$\text{maximize:} \quad v_T - w_T \qquad (4\text{-}1)$$

$$\text{subject to:} \quad -\sum_{j=1}^{n} a_{0j}x_j + v_0 - w_0 \leq M_0 \qquad (4\text{-}3)$$

$$-\sum_{j=1}^{n} a_{tj}x_j + v_t - w_t - v_{t-1}(1+r_\ell) + w_{t-1}(1+r_{bt-1}) \leq M_t \qquad (4\text{-}2)$$

$$t = 1,2,\ldots, T$$

$$-E_f r_{bt} + bw_t \leq -E_f a \qquad t = 0,1,\ldots, T \qquad (4\text{-}29)$$

$$0 \leq x_j \leq 1 \qquad j = 1,2,\ldots, n \qquad (4\text{-}12)$$

$$v_t, w_t, r_{bt} \geq 0 \qquad t = 0,1,\ldots, T \qquad (4\text{-}10)$$

where E_f is the fixed equity. It should be noted that this formulation results in a bi-linear problem.

The implication of fixing the equity at a constant level throughout the planning period makes the problem lose generality in a certain sense, considering that the equity is not generated by the model itself. This situation can represent the case of a firm that starts with an initial equity that remains constant during the planning period, where all earnings provided by the projects are invested in other opportunities outside the firm. In considering this problem, the funds provided by the proposals do not have to satisfy any equity constraints, and the amounts borrowed are restricted only according to the debt-equity ratio determined by the model.

Optimality Analysis for the Modified Nonlinear Programming Problem

In a previous section there was demonstrated the concavity of the objective function (4-1), and the convexity of the budget (4-2) and project upper bound constraints (4-8). There remains the modified interest rate constraint (4-7), which is transformed into a linear function with the assumption made about the equity. This constraint can also be considered convex. Thus, for this modified nonlinear programming problem there exists an optimal point $\underline{x}^*$ if it satisfies Equations (4-16) through (4-21), the Kuhn-Tucker necessary conditions. Furthermore, the sufficient conditions for optimality are satisfied so any such point can be considered a global optimal point. In the next chapter this problem is solved and the results are compared with the solution given by the general problem.

Linear Programming Problem

Now there are discussed the assumptions followed in establishing a simplified approach to the nonlinear model. Observing Equations (4-2), (4-5), (4-7), and (4-11), it is noted that if r_{bt} is fixed the constraints become linear. Taking that into account and deleting Equation (4-7), there can be suggested an iterative process for solving the linear programming problem: given at each iteration a value of interest rate on borrowed money calculated by Equation (4-6) (as an equality), solve the linear programming problem to obtain the values of debt and equity. These are then re-entered into Equation (4-6) and the LP solved again. The process stops when the interest rate vector converges. Figure 4-1 shows a flowchart of the algorithm.

A difficulty arises when the equity is zero. The model itself does not seem to solve this situation. One possibility is to assign an infinitely large value to the interest rate when this occurs in order to force the debt to zero. It was demonstrated computationally that this approach does not converge. The difficulty is avoided by defining the debt-equity ratio as the average of debt over

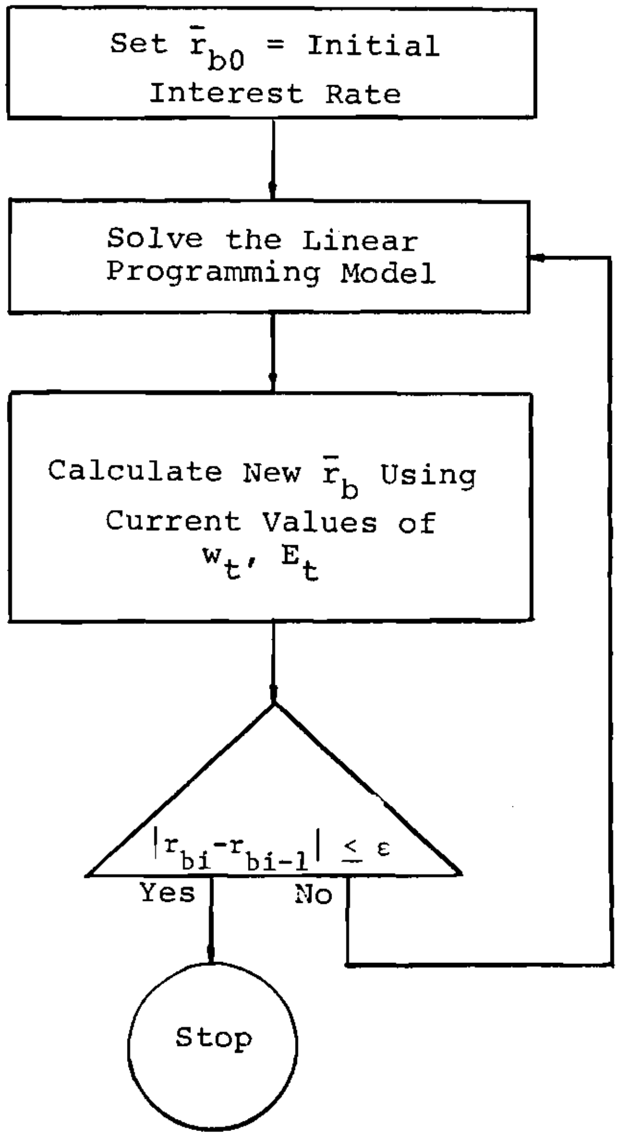

Figure 4-1. Flow Chart of Iterative Process Using Linear Programming Solutions.

the average equity during the planning period. This average can be weighted average or not, depending on the cash flows generated by the solution at different interest rates.

The model can be written as:

$$\text{maximize:} \quad v_T - w_T \tag{4-1}$$

$$\text{subject to:} \quad -\sum_{j=1}^{n} a_{0j}x_j + v_0 - w_0 \le M_0 \tag{4-3}$$

$$[u_{Mt}] \quad -\sum_{j=1}^{n} a_{tj}x_j + v_t - w_t - v_t(1+r_\ell) + w_{t-1}(1+\bar{r}_b) \ge M_t \qquad t = 1,2,\ldots, T \tag{4-2}$$

$$E_1 - \sum_{j=1}^{n} a_{1j}x_j - v_0 r_\ell + w_0\bar{r}_b \le E_0 \tag{4-30}$$

$[u_{Et}]$

$$E_t - E_{t-1} - \sum_{j=1}^{n} a_{tj}x_j - v_{t-1}r_\ell + w_{t-1}\bar{r}_b \le 0 \qquad t = 2,3,\ldots, T \tag{4-31}$$

$$[u_{xj}] \quad x_j \le 1 \qquad j = 1,2,\ldots, n \tag{4-8}$$

$$x_j \ge 0 \qquad j = 1,2,\ldots, n \tag{4-10}$$

$$v_t,\ w_t,\ E_t \ge 0 \qquad t = 0,1,\ldots, T \tag{4-9}$$

Here, $\bar{r}_b$ is obtained at each stage by

$$\bar{r}_b = a + b \left(\frac{\sum_t w_t f_t}{\sum_t E_t f_t}\right) \quad (4\text{-}32)$$

where f_t is the weight given to time t.

Optimality Analysis for the LP Problem

For every linear programming problem there is associated a dual linear programming problem which satisfies certain properties used in obtaining the solution of the original problem. Utilizing the definition of duality the primal problem is transformed into the dual as follows.[19]

$$\text{minimize:} \quad \sum_{t=0}^{T} u_{Mt} M_t + u_{E1} E_0 + \sum_{j=1}^{n} u_{xj} \quad (4\text{-}33)$$

$$\underset{(x_j)}{\text{subject to:}} \; - \sum_{t=0}^{T} a_{tj} u_{Mt} + u_{xj} - \sum_{t=1}^{T} a_{tj} u_{Et} \leq 0 \quad (4\text{-}34)$$

$$u_{Mt} - (1+r_\ell) u_{M,t+1} - r_\ell u_{E,t+1} \leq 0$$

$$(v_t) \qquad t = 0,1,\ldots, T-1 \quad (4\text{-}35)$$

$$u_{MT} \leq 1 \quad (4\text{-}36)$$

$$-u_{Mt} + (1-\bar{r}_b) u_{M,t+1} + \bar{r}_b u_{E,t+1} \leq 0$$

$$(w_t) \qquad t = 0,1,\ldots, T-1 \quad (4\text{-}37)$$

$$-u_{MT} \leq -1 \quad (4\text{-}38)$$

$$u_{Et} - u_{E,t+1} \leq 0 \qquad t = 1,2,\ldots, T-1 \tag{4-39}$$

(E_t)

$$u_{ET} \leq 0 \tag{4-40}$$

Equation (4-34) is derived from the project activity variables, Equations (4-35) and (4-36) from lending activities, Equations (4-37) and (4-38) from borrowing activities, and Equations (4-39) and (4-40) from equity variables.

From Equations (4-36) and (4-38) it can be established that $u^*_{MT} = 1$

From Equations (4-35) and (4-37) one obtains the following expressing:

$$(1+\bar{r}_b)u_{M,t+1} + \bar{r}_b u_{E,t+1} \leq u_{Mt} \leq (1+r_\ell)u_{M,t+1} + r_\ell u_{E,t+1} \tag{4-41}$$

If only borrowing activities are performed at time t, the left side of (4-41) has to be satisifed as equality:

$$(1+\bar{r}_b)u^*_{M,t+1} + \bar{r}_b u^*_{E,t+1} = u^*_{Mt} \tag{4-42}$$

Rearranging terms the following expression is achieved:

$$(1+\bar{r}_b) = \frac{u^*_{Mt} + u^*_{E,t+1}}{u^*_{M,t+1} + u^*_{E,t+1}} \qquad t = 0,1,\ldots, T-1 \tag{4-43}$$

Similarly, if $v_t^* > 0$ the right side of (4-41) becomes an equality:

$$u^*_{Mt} = u^*_{M,t+1}(1+r_\ell) + u^*_{E,t+1}r_\ell \tag{4-44}$$

Rearranging terms:

$$(1+r_\ell) = \frac{u^*_{Mt} + u^*_{E,t+1}}{u^*_{M,t+1} + u^*_{E,t+1}} \qquad t = 0,1,\ldots, T-1 \qquad (4\text{-}45)$$

These results are similar to those obtained by Weingartner for the basic horizon model. Equations (4-43) and (4-45) differ from the classical formulas by the inclusion of the $u_{E,t+1}$ terms.

Analogous results are obtained for project acceptance criteria. From Equation (4-34) one has:

$$u^*_{xj} \le \sum_{t=1}^{T} a_{tj}u_{Et} + \sum_{t=0}^{T} a_{tj}u_{Mt} \qquad j = 1,2,\ldots, n \qquad (4\text{-}46)$$

For completely accepted projects $x^*_j = 1$, $u^*_{xj} > 0$ and

$$u^*_{xj} = \sum_{t=1}^{T} a_{tj}u_{Et} + \sum_{t=0}^{T} a_{tj}u_{Mt} \qquad (4\text{-}47)$$

Similarly, if the proposal j is partially accepted $u_{xj}{}^* = 0$ and:

$$0 \le \sum_{t=1}^{T} a_{tj}u_{Et} + \sum_{t=0}^{T} a_{tj}u_{Mt} \qquad (4\text{-}48)$$

Thus, the pricing of the cash flows to determine project acceptance must be performed with the dual variables from the cash balance constraints and the equity constraints. Since the u_{Et} are not known until the problem is solved, one cannot develop a simple acceptance criterion based on a common interest rate for borrowing-lending, as is the case for the basic horizon model.[24]

In this chapter there were presented and analyzed three models for deterministic capital budgeting. The following chapter presents computationsl experience for these models using some sample project sets.

CHAPTER V

COMPUTATIONAL RESULTS

In this chapter are presented the computational results for the three basic approaches developed in the previous chapter:

1. Non-convex nonlinear programming problem
2. Convex nonlinear programming problem
3. Linear programming problem

These are referred throughout the chapter as non-convex, convex, and linear problems respectively.

Three sets of projects were selected for solution by these approaches. A horizon of seven years was assumed for each set, and the projects were generally defined so that there would be large disbursements during the initial years and large revenues near the horizon. The result was a tendency to borrow in almost all the periods. Following are shown the project sets:

Set A. This is composed of four projects; there is assumed a lending interest rate of 4.0 percent. The relationship between the borrowing interest rate and the debt-equity ratio is $IR_i = 0.0531 + 0.0378\ (w_i/E_i)$, and an initial equity equal to $50,000 is assumed.

Set B. This set consists of ten projects, with a lending interest rate is 4.0 percent. The same relationship between borrowing interest rate and debt-equity ratio is used as for set A, that is, $IR_i = 0.0531 + 0.0378\ (w_i/E_i)$, the initial equity is also \$50,000. The cash flows and budget for each period of time, for sets A and B are given by Tables 5-1 and 5-2.

Table 5-1. Cash Flows and Budgets for Set A.

t	a_{t1}	a_{t2}	a_{t3}	a_{t4}	M_t
0	-1,000	-2,000	-1,000	-5,000	1,000
1	-2,000	-2,000	-5,000	-5,000	5,000
2	-5,000	3,000	-6,000	-2,000	300
3	-5,000	-2,000	4,000	-2,000	500
4	-5,000	-2,000	-8,000	-	500
5	10,000	4,000	-7,000	-500	-
6	10,000	4,500	20,000	15,000	500
7	10,000	5,000	20,000	20,000	1,000

Set C. In this set the cash flows and budget for each period of time and the initial equity are the same as for set B. Otherwise, the lending interest rate is assumed to be 5.5 percent and the relationship of debt/equity ratio-borrowing interest rate is considered as $IR_i = 0.0679 + 0.023595\ (w_i/E_i)$.

Overview of Solution Procedures

For solving nonlinear programming problems a large number of techniques have been developed. Due to the many

Table 5-2. Cash Flows and Budgets for Set B.

t	a_{t1}	a_{t2}	a_{t3}	a_{t4}	a_{t5}	a_{t6}	a_{t7}	a_{t8}	a_{t9}	a_{t10}	M_t
0	-1000	-2000	-5000	-1000	-	-500	-2000	-2000	-10000	-10000	1000
1	-2000	-2000	-1000	-1000	-	-500	-1000	-2000	-10000	-	5000
2	-10000	-2000	-	-1000	-20000	-500	1000	2000	-	-	300
3	-20000	-3000	-1000	1000	-20000	-500	1000	2000	-10000	-	500
4	1000	-10000	-	1000	5000	-5000	1000	2000	-10000	-	500
5	10000	10000	-5000	2000	-5000	1500	1000	-2000	-20000	-2000	-
6	10000	40000	-5000	-5000	25000	1500	1000	2000	60000	15000	500
7	-50000	-2000	35000	12000	5000	10000	500	-	50000	25000	1000

forms that a nonlinear programming problem can take, specific techniques have been developed for solving special problems. Only a few procedures have proved to be useful in solving general nonlinear programming problems. Several procedures employ penalty function techniques which require the use of derivatives. Other search methods utilize a more simplified approach that gives good results for certain types of problems.[18]

An indirect technique that can be suggested is to fix certain variables so as to linearize the nonlinear constraints. Then an LP problem is solved. In a second problem there are made variable the terms that were fixed and fixed those which were variable in the first problem. The procedure is repeated until no change in the variables is reached.

Non-Convex Nonlinear Programming Problem

Hooke and Jeeves Solution Algorithm

One way of solving the non-convex problem is to apply a nonlinear programming algorithm directly. The program Comet,[19] which utilizes a penalty function technique, was used with unsatisfactory results, so that other procedures were attempted.

Other algorithms for solving nonlinear programming problems which do not employ derivatives are the direct search methods. A procedure for an n-variable optimization problem would be to fix (n-1) variables at initial values, and search over the n^{th} decision variable until a maximizing (minimizing) solution is found with respect to that one variable. The process is repeated for each of the (n-1) remaining variables until no change in the objective function value is obtained. A method which uses this principle is the Hooke and Jeeves algorithm. This gave good results in solving this type of nonlinear programming problem. The method has two phases: the first is an exploratory search phase which establishes a direction of improvement, and the second is a pattern move which changes the current solution vector to another point in the solution space. The algorithm can be summarized as follows:

a. A base point is selected and the objective function is evaluated.

b. Local searches are made in the directions $x_i + S_i$ and $x_i - S_i$, evaluating the function to see if an improvement is achieved.

c. If there is no improvement in the function value, the step size S_i is reduced and searches are made from the previous base point.

d. If the value of the function has improved, a temporary base $x_{io}^{(k+1)}$, is established by the following expression:

$$x_{io}^{(k+1)} = x_i^{(k+1)} + \alpha(x_i^{(k+1)} - x_i^{(k)}) \qquad (5\text{-}1)$$

where i = variable index

α = acceleration factor

k = stage index (a stage is the end of n searches and subscript o denotes the temporary base).

e. If the temporary base results in an improvement in the function value, a new local search is performed about the temporary base, a new base is located, and the value of the function checked. The process is continued as long as the function improves.

f. If the temporary base does not give an improvement in the function value, a search is made from the previous best point.

g. The procedure is terminated when the convergence criterion is satisfied.

A diagram for the Hooke and Jeeves search is shown in Figure 5-1.

The Hooke and Jeeves algorithm was originally designed for solving multivariable unconstrained, nonlinear functions. It can be used for solving constrained nonlinear programming problems by evaluating the function subject to the constraints after each movement. In the case of the non-convex problem, Equations (4-1) through (4-3), (4-5), (4-7), and (4-10) through (4-12), it is necessary to define which terms will be considered as decision variables. Observing the structure of the problem, it can be suggested that the borrowing interest rates be considered as variables to be changed by the Hooke and Jeeves algorithm. First, by fixing those variables the constraints (4-2), (4-5), (4-7), and (4-11) are transformed

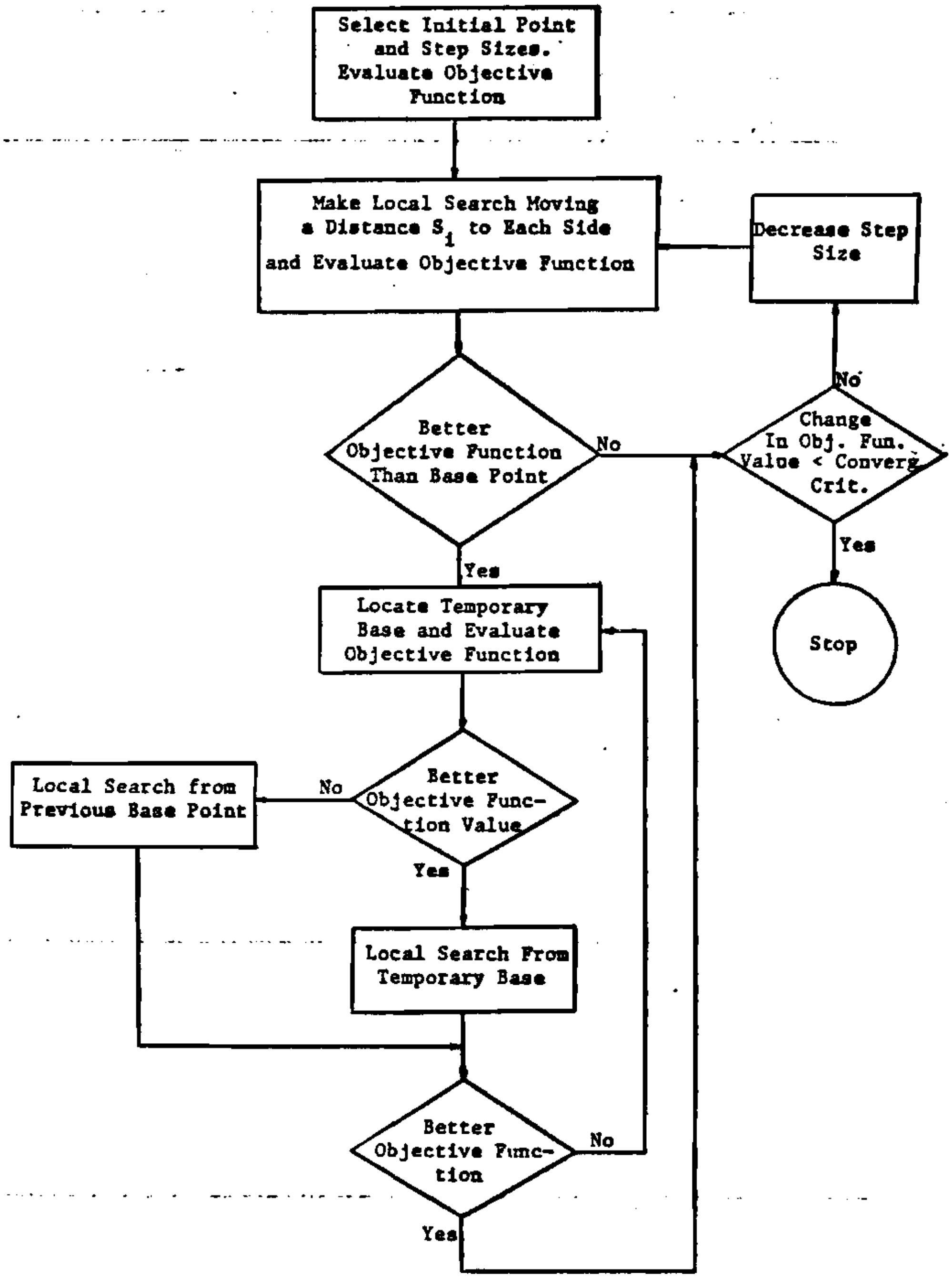

Figure 5-1. Flow Chart of Hooke and Jeeves Algorithm.

into linear constraints, so that instead of evaluating a nonlinear function as in the original Hooke and Jeeves algorithm, there is solved an LP problem. The objective function value of the LP is considered in the convergence criterion of the overall algorithm. Second, because the range of these variables is narrower than that for the lending and borrowing activities, for example, it is presumed that they are more easily managed.

There are four basic parameters to be defined in order to utilize the Hooke and Jeeves algorithm:

ALPHA = factor for extending the size of the initial steps, $\alpha \geq 1$.

BETA = factor for reducing the initial step size, $0 \leq$ BETA ≤ 1.

EPSY = error in objective function to be reached before program terminates (difference between current value and previous stage value).

S_i = vector of initial step-sizes to be used for each of the variables.

Results of the Non-Convex Nonlinear Programming Model

In solving the non-convex problem it was assumed ALPHA = 2.0, BETA = 0.5, S_i = 0.5 (the initial value of the variable), and EPSY = 5.0 for sets A and B. For set C the same values were used except EPSY = 25.0.

By the convexity features of the problem, it cannot be guaranteed that a global optimal solution will be obtained.

Thus, as many as ten different starting solutions were tried. There were obtained similar objective function values for all the set of initial points. In addition, the final values of the variables are very similar. The algorithm itself is a numerical method, so that some errors are incurred, first, due to the values of the parameters and the convergence criterion employed, and second, due to round off errors involved in the solution of the LP problems.

Tables A-1, A-2, and A-3 show the objective function value, the borrowing interest rate obtained for each period of time, the number of function evaluations, the solution computer time, the initial interest rate used in each case, and the units of accepted projects for sets A, B, and C, respectively.

Observing the objective function values, one can calculate an error with respect to the best solution of 0.11 percent for set A, 0.24 percent for set B, and 0.42 percent for set C. These are small considering the numeric features of the algorithm.

Observing Tables A-4 through A-6, it can be noted that for the three sets, the equity shows a decreasing pattern with respect to time, which is opposite to the pattern shown by the debt and the borrowing interest rate, which are increased from time zero to the fifth period. In the last two periods, they have values of zero and a (intercept of regression equation), respectively, except for cases A.1.1, A.2.1, A.4.1, A.6.1 to A.8.1, and A.10.1. A typical result for set A is given in Table 5-3.

Table 5-3. Typical Results for Set A Given by the Non-Convex Model.

t	v_t	w_t	E_t	w_t/E_t	r_{bt}
0	-	6696	50000	0.1339	0.0582
1	-	12459	38814	0.3209	0.0652
2	-	16118	34528	0.4668	0.0708
3	-	18308	31605	0.5792	0.0750
4	-	26646	21476	1.2407	0.1000
5	-	30051	17149	1.7523	0.1193
6	-	70	21639	0.0032	0.0532
7	39493	-	-	-	0.0531
$x_1 = 0.037$		$x_2 = 1.000$	$x_3 = 0.670$	$x_4 = 1.000$	

From Equation (4-7):

$$E_t(a-r_{bt}) + bw_t \leq 0 \qquad t = 0,1,\ldots, T$$

It can be deduced that when $E_t = 0$, necessarily $w_t = 0$. Otherwise, when $w_t = 0$, E_t may be greater than or equal to zero. This last fact can be seen in all the cases for set B and C. It can also be observed that there are lending activities in periods of time where no borrowing activities are presented.

Figure 5-2 shows how the objective function value changes as a function of the number of objective function

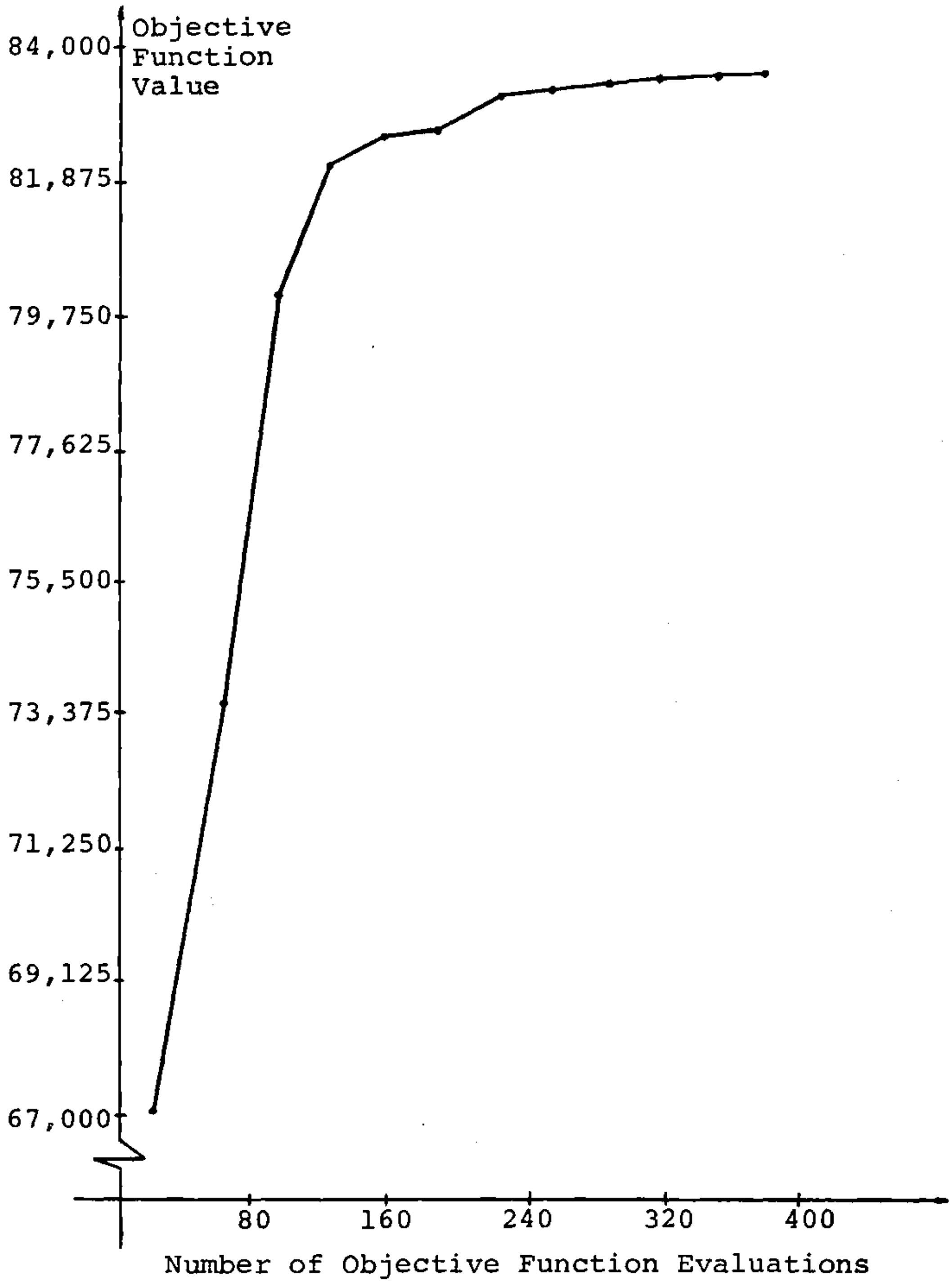

Figure 5-2. Non-Convex Model, Convergence for Set C.

evaluation, for the case C.5.1. (Tables A-3 and A-9).
It is observed that the closer the values approach the optimal point, the slower the absolute convergence.

Optimality Analysis for the Non-Convex Problem: Kuhn-Tucker Conditions

For the non-convex nonlinear programming problem, Equations (4-1) through (4-3), (4-5), (4-7), and (4-10) through (4-12), $\underline{x}^*$ is a Kuhn-Tucker point if there exist $u_1, u_2, \ldots, u_m$, such that $\underline{x}^*$ satisfies Equations (4-16) through (4-21). Equations (4-17) and (4-19) represent complementary slackness between the variables and constraints of the original problem and the Kuhn-Tucker multipliers and constraints. Equation (4-16) yields the following set of equations:

For projects:

$$\sum_{t=0}^{T} a_{tj}\, u_{Mt} + \sum_{t=1}^{T} a_{tj}\, u_{Et} \leq u_{xj} \qquad j = 1,2,\ldots, n \tag{5-2}$$

For lending:

$$-u_{Mt} + (1+r_{\ell})u_{M,t+1} - r_{\ell}u_{E,t+1} \leq 0 \qquad t = 0,1,\ldots, T-1 \tag{5-3}$$

$$-u_{MT} + 1 \leq 0 \tag{5-4}$$

For borrowing:

$$u_{Mt} - (1+r_{bt})u_{M,t+1} - r_{bt}u_{E,t+1} - bu_{it} \leq 0 \tag{5-5}$$

$$t = 0,1,\ldots, T-1$$

$$u_{MT} - bu_{iT} - 1 \leq 0 \tag{5-6}$$

For equity:

$$-u_{Et} + u_{E,t+1} - (a-r_{bt})u_{i,t-1} \leq 0 \qquad t = 1,2,\ldots, T-1 \tag{5-7}$$

$$-u_{ET} - (a-r_{bT})u_{iT} \leq 0 \tag{5-8}$$

For interest rate:

$$-w_t u_{M,t+1} - w_t u_{E,t+1} + E_t u_{it} \leq 0 \qquad t = 0,1,\ldots, T-1 \tag{5-9}$$

$$-E_T u_{iT} \leq 0 \tag{5-10}$$

The Kuhn-Tucker multipliers are not given by the nonlinear algorithm so that is necessary to solve all the Kuhn-Tucker conditions in order to obtain the values. This was done by first fixing certain multipliers using complementary slackness conditions, and then solving for the rest using Gauss-Jordan substitution. Table 5-4 shows the multiplier values for case A.7.1.

Convex Nonlinear Programming Problem

Iterative Solution Procedure Adapted to Convex Nonlinear Programming Model

A procedure that can be suggested for solving the convex nonlinear programming problem, Equations (4-1) through (4-3), (4-10), (4-12), and (4-29), is to fix certain variables in order to linearize the nonlinear epxressions. Then one can

Table 5-4. Multiplier Values for Case A.7.1.

	Budget Constraints	Equity Constraints	Interest Rate Constraints	Project Constraints
t	u_{MT}	u_{Et}	u_{it}	u_{xj}
0	2.1278	0.16423	0.26800	0
1	2.0012	0.16094	0.64373	5123.6
2	1.8457	0.15044	0.85796	0
3	1.6829	0.13347	0.96831	5573.1
4	1.5209	0.09488	1.76231	
5	1.2899	0.	2.02730	
6	1.0748	0.	0.28594	
7	1.0000	0.	0.	

solve an LP problem (step 1). In a subsequent problem (step 2) we can make variable the terms that were fixed and fix those which were variable in step 1.

For the convex problem this two-step procedure can be applied as follows: Fixing the borrowing interest rate, r_{bt}, step 1 becomes:

$$\underset{x_j,\ v_t,\ w_t}{\text{maximize:}} \quad v_T - w_T \tag{4-1}$$

$$\text{subject to:} \quad -\sum_{j-1}^{n} a_{0j}x_j + v_0 - w_0 \leq M_0 \tag{4-3}$$

$$-\sum_{j-1}^{n} a_{tj}x_j + v_t - v_{t-1}(1+r_\ell) - w_t + w_{t-1}(1+r_{bt-1}) \leq M_t \qquad (4\text{-}2)$$

$$t = 1,2,\ldots, T$$

$$b_{wt} \leq -E_f(a-r_{bt}) \qquad t = 0,1,\ldots, T \qquad (5\text{-}11)$$

$$0 \leq x_j \leq 1 \qquad j = 1,2,\ldots, n \qquad (4\text{-}12)$$

$$v_t,\ w_t,\ x_j \geq 0 \qquad t = 0,1,\ldots, T \qquad (5\text{-}12)$$

where E_f and r_{bt} are fixed.

In step 2, we fix w_t and solve:

$$\underset{x_j,\ v_t,\ r_{bt}}{\text{maximize:}}\ v_T \qquad (5\text{-}13)$$

$$\text{subject to:}\quad -\sum_{j=1}^{n} a_{0j}x_j + v_0 - w_0 \leq M_0 \qquad (4\text{-}3)$$

$$-\sum_{j=1}^{n} a_{tj}x_j + v_t - v_{t-1}(1+r_\ell) + w_{t-1}r_{bt-1} \leq M_t + w_t - w_{t-1} \qquad (4\text{-}2)$$

$$t = 1,2,\ldots, T$$

$$-E_f r_{bt} \leq -aE_f - bw_t \qquad t = 0,1,\ldots, T \qquad (5\text{-}14)$$

$$0 \leq x_j \leq 1 \qquad j = 1,2,\ldots, n \qquad (4\text{-}12)$$

$$v_t,\ r_{bt} \geq 0 \qquad t = 0,1,\ldots, T \qquad (5\text{-}15)$$

The two-step process is performed until no change in the values of the variables is obtained. Set A was solved by this approach, employing different initial solutions, r_{bt}, with the results shown in Figure 5-3.

The axis $\underline{r}^{\circ}_{bt}$ represents a starting interest rate which is the same for all time points t = 0,1,..., T. It can be noted that the procedure reaches the same optimal point for only certain initial solutions, so that it does not seem to be a general procedure for solving the problem. Accordingly, this method was abandoned.

Results of the Hooke and Jeeves Algorithm for the Convex Nonlinear Programming Problem

In this problem the values of ALPHA, BETA, EPSY, and S_i are the same as in the case before. Again, different starting solutions were used to see if the same optimal point is achieved. Tables A-7 through A-12 show the results for this problem. It can be noted that a similar optimal point is reached for all the different initial solutions, proving that the algorithm is capable of solving this type of nonlinear programming problem and obtaining the global optimum.

The pattern followed by the optimal values of the variables in this problem is similar to those shown for the non-convex NLP problem. A typical result for set A is given in Table 5-5. Due to the formulation itself, the equity remains constant over the planning period. Otherwise, the debt and borrowing interest rate increase up to fifth period, when they reach their maximum values. In period seven their

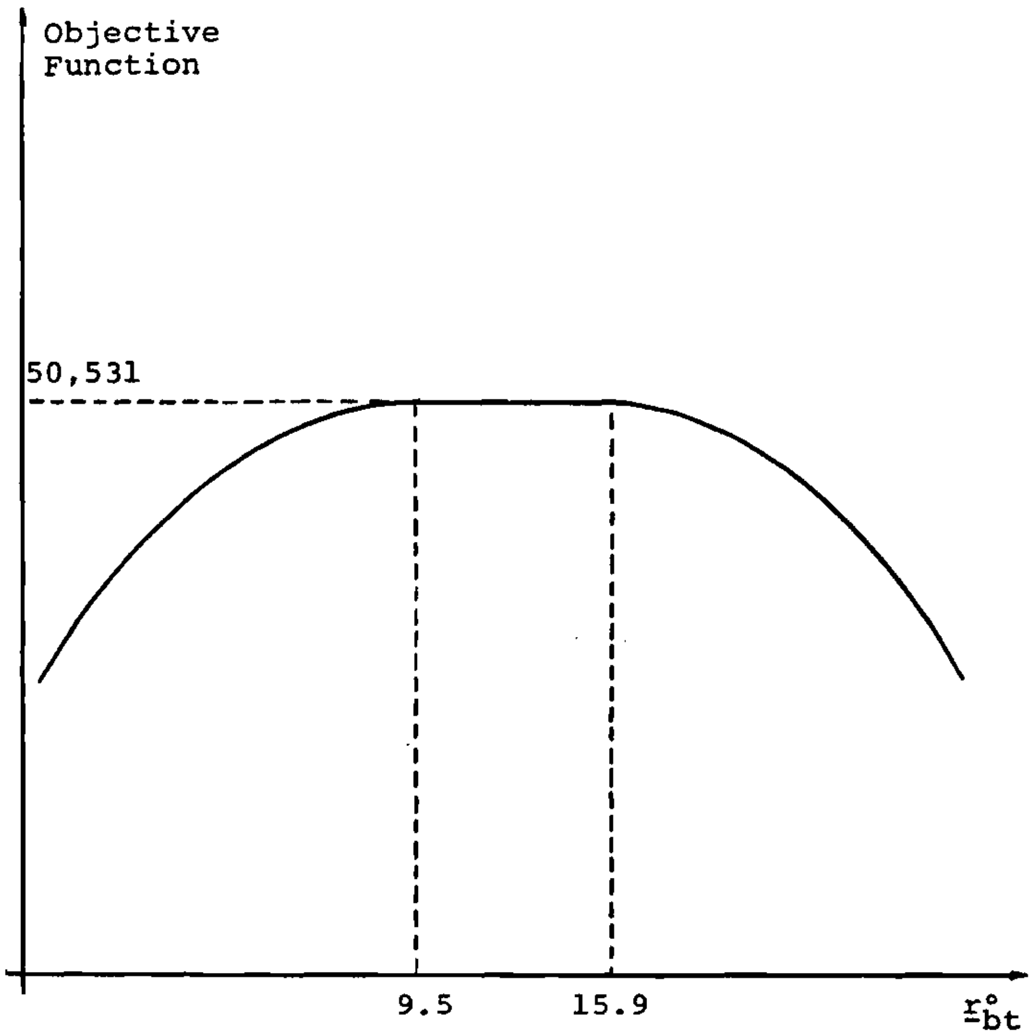

Figure 5-3. Results for Set A Given by the Two-Step Procedure Developed for Solving the Convex Model.

values are reduced drastically: a zero value for debt and a value of a (intercept of the regression equation) for the borrowing rate at time T.

Table 5-5. Typical Result for Set A Given by the Convex Model.

t	v_t	w_t	E_t	w_t/E_t	r_{bt}
0	-	7999	50000	0.1599	0.0591
1	-	17472	50000	0.3494	0.0663
2	-	28331	50000	0.5666	0.0745
3	-	34941	50000	0.6988	0.0795
4	-	52219	50000	1.0443	0.0925
5	-	50555	50000	1.0111	0.0913
6	-	5173	50000	0.1035	0.0570
7	50530	-	50000	-	0.0531

$x_1 = 1.000$ $x_2 = 1.000$ $x_3 = 1.000$ $x_4 = 1.000$

Lending activities are performed only in period T. The maximum difference in the objective function values can be calculated as 0.01 percent for set A, 0.3 percent for set B, and 0.48 percent for set C, which are not very significant considering the numerical characteristics of the algorithm.

Figure 5-4 shows how the objective function values change with the number of objective function evaluations by the algorithm in solving the problem for case C.4.2.

Results of the Linear Programming Model

For solving this problem, Equations (4-1) through (4-3),

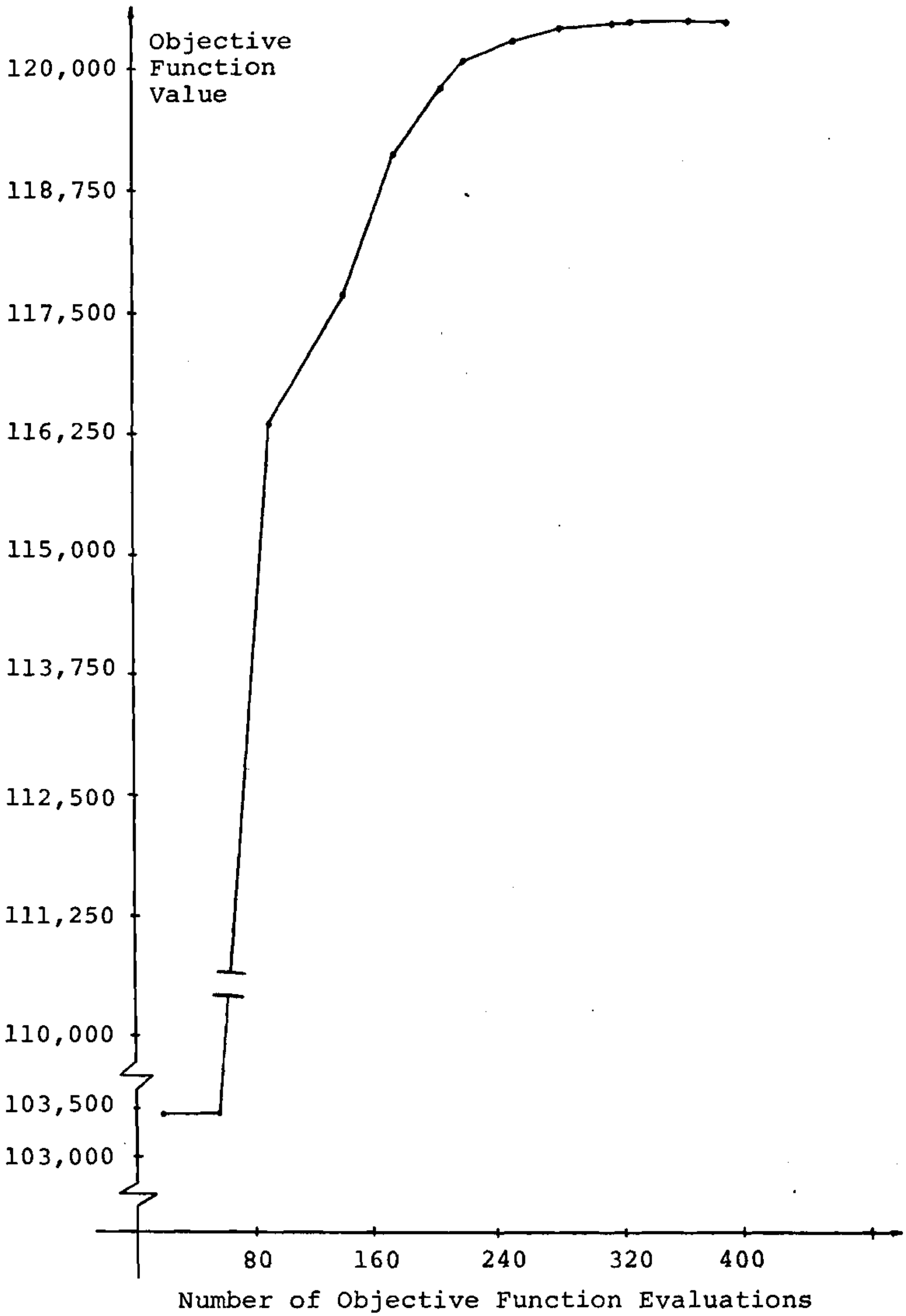

Figure 5-4. Convex Model, Convergence for Set C.

(4-8) through (4-10), (4-30), and (4-31), it was assumed $f_t = (T+1)^{-1}$ so that the same weight is given to debt and equity for each period of time in the interest rate equation:

$$\bar{r}_b = \left(\frac{\sum_t w_t f_t}{\sum_t E_t f_t}\right) \tag{5-16}$$

Table 5-6 gives the results from applying LP model to set A:

Table 5-6. Result for Set A Given by the LP Model.

t	v_t	w_t	E_t
0	-	7865	50000
1	-	17339	35525
2	-	28006	24559
3	-	34480	17585
4	-	51565	-
5	-	51280	285
6	-	7460	-
7	46481	-	52951
	x_1 = 0.865	x_2 = 1.000	
	x_3 = 1.000	x_4 = 1.000	

There was obtained an average interest rate $\bar{r}_b = 0.094$, with a debt-equity ratio equal to 1.095. It can be noted that the borrowing and lending activities follow the same pattern as in the nonlinear models while the equity does not.

In the next chapter several conclusions are reached about the results obtained, and there are discussed the implications of each model.

Evaluation

Comparison Among Models

Analyzing the results, it is interesting to observe that the amounts borrowed by the convex model are larger than for the non-convex model. This occurs because interest paid on borrowed money decreases equity in the non-convex model but not in the convex model. The lower equity leads to higher interest rates and less borrowing in the non-convex model. Also, the equity constraints themselves limit the feasible space; these constraints are absent in the convex model. The larger amounts borrowed are invested in profitable projects, so that larger objective function values are achieved. These results cannot be generalized because the equity is fixed in the convex model, and the two models are solving different problems. A low fixed equity could lead to a lower objective function for the convex model.

Comparing the non-convex with the linear model, the latter results in more borrowing but also in more cash at the horizon (objective function value). This can be explained as follows: Both models use the same interest rate function, but the linear model computes an average debt-equity ratio where the debt and equity values for each time period receive equal weight:

$$\text{ratio}_1 = \frac{\sum_t w_t f_t}{\sum_t E_t f_t} \tag{5-17}$$

where $f_t = 1/(1+T)$.

The non-convex model, on the other hand, computes a debt-equity ratio w_t/E_t each time period. In a sense, the non-convex model can be interpreted as determining an "average" debt-equity ratio, except that here the average would be defined by a different formula:

$$\text{ratio}_2 = \left(\frac{1}{\sum_t f_t}\right) \sum_t \left(\frac{f_t w_t}{E_t}\right) \tag{5-18}$$

In this case the f_t weights correspond to the amounts borrowed each period. The averaging process defined by (5-18) results in a higher ratio than (5-17) for the example problem.

Also, the equity constraints in the non-convex model define a smaller feasible space than is the case for the LP model. The equity actually approaches zero values for set A using the non-convex model (Table A-1).

Comparison with Basic Horizon Model

It is instructive to compare the results achieved by the nonlinear models with those of Weingartner's basic horizon model, which assumes a fixed borrowing rate (Table A-13). Taking set A, for example, Table 5-7 show the results given by the basic horizon model at four different interest rates.

It can be noted that the amounts borrowed are greater

Table 5-7. Results for Set A Given by the Basic Horizon Model.

r_{bt}	0.0531		0.075		0.100		0.125	
t	w_t	v_t	w_t	v_t	w_t	v_t	w_t	v_t
0	8000	-	8000	-	8000	-	8000	-
1	17420	-	17600	-	17800	-	18000	-
2	28050	-	28620	-	29280	-	29950	-
3	34040	-	35270	-	36710	-	38190	-
4	50350	-	52410	-	54880	-	57470	-
5	46520	-	49840	-	53870	-	58150	-
6	-	10090	3581	-	9253	-	15420	-
7	-	57050	-	52150	-	45820	-	38650
x_1	1.000		1.000		1.000		1.000	
x_2	1.000		1.000		1.000		1.000	
x_3	1.000		1.000		1.000		1.000	
x_4	1.000		1.000		1.000		1.000	

than for the convex model and hence for the non-convex model, due to the fact that no restrictions are made in the way of obtaining funds. Note that this model accepts all projects completely, whereas the nonlinear models are more selective, reflecting the economics of borrowing and the smaller feasible space.

Other Considerations

Observing the borrowing interest rates obtained by nonlinear models one can observe rapid changes in the last periods of the planning period. The borrowing interest rates are a function of the existing prime rate, the financial characteristics of the borrower, and the overall business volume of the borrower at the bank.[22] Most corporate debt consists of long-term bonds and other debt instruments. Once issued, the interest rate on the debt instrument remains fixed. The models presented all assume annual debt instruments and thus predict much more rapid changes in aggregate interest payments than would actually occur.

The use of the LP model, which employs a weighted average interest rate, seems more suitable for such slower changes in actual rates. Alternatively, long-term debt variables can be explicitly included in the nonlinear models, with an increase in model size.

Computer Requirements

Considering a commercial cost of $1,000/hour of CPU time in Tables 5-8, there are shown average costs for solving the nonlinear models. The program requires about 22,000 words core space (CDC).

Chapter Summary

In this chapter were presented the computational results for the three approaches developed in Chapter IV: the non-convex, convex, and linear problems. First is given an overview of solution procedures. Second, there is discussed the Hooke and Jeeves algorithm and how it is adapted

Table 5-8. Average Time and Cost for Solving the Nonlinear Models.

	Non-Convex Model			Convex Model		
	Set A	Set B	Set C	Set A	Set B	Set C
Avg. Time* (Min. CPU)	4.633	11.383	8.767	3.100	8.000	6.700
Avg. Cost,$	77.216	189.717	146.117	51.667	133.333	111.667

*CDC Cyber 74

for solving the non-convex problem. These results are then shown, followed by an optimality analysis using the Kuhn-Tucker conditions. Afterward, an attempt is made to solve the convex problem by an iterative linear procedure, which is abandoned. The convex problem is solved by the Hooke and Jeeves algorithm and the results are presented. The third approach consists of an iterative LP procedure, and results are given for this. Finally, the results of different models are compared with one another and with results obtained by the basic horizon model.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

Most of the formulations of capital budgeting models consider the interest rate on borrowed money assuming a constant rate with fixed debt limits, or by establishing a rising supply curve for funds where higher interest rates are associated with successive amounts borrowed. It is not considered in those models that the lender of funds takes into account, among other factors, the capital sturcture of the firm in determining its borrowing ability.

The purpose of this research was to extend a capital budgeting model, taking Weingartner's basic horizon model as a point of departure and to include borrowing interest rate as a function of the debt-equity ratio. Other goals were to demonstrate the computational ability of an algorithm for solving the extended model, and to obtain economic interpretations.

Three models were developed, and these were solved using nonlinear and linear programming techniques. The general non-convex model can be related to the situation of a firm which starts its operations with an initial equity, where there is no allowance for payment of dividends and for issuance or purchase of stock. The equity can only be affected by the outlays and returns from investment proposals selected and by the interest on lent or borrowed money. The expression which defines the equity restricts the problem in such a way that those constraints must be satisfied by the funds generated by the proposals. The model itself can be used in the planning activity of a small firm which knows the cash flows to be generated by the set of proposals, may be in a captive market, and due to management policy or restrictive covenants does not pay dividends during the planning period. The non-convex model was solved using the Hooke and Jeeves algorithm. Basically, the Hooke and Jeeves algorithm is employed for solving multivariable unconstrained, nonlinear functions. It can be used for solving a constrained nonlinear programming problem by evaluating the function subject to the constraints, resulting in LP subproblems.

The basic feature of the second model, a convex nonlinear model is in the assumption of a fixed equity. Again, there is no payment of dividends, issuance or purchase of stock. All funds generated by the proposals are invested in other outside opportunities after servicing the debt, and do not have to satisfy any restriction within the firm. This model is also solved by the Hooke and Jeeves algorithm.

The third model, an iterative linear model can be used as a planning tool where the assumption of an average interest rate is appropriate. This might reflect the situation of a firm which does not change drastically its financial structure. The average interest rate is determined by a weighted debt-equity ratio.

After analyzing the results presented in the previous chapter some comments should be made:

1. As a first point it can be said that after trying with different starting points for the non-convex nonlinear programming model the solutions achieved are essentially the same. In some way, the nonconvexity of the set of constraints does not appear to result in different optima, at least in the range of practical values.
2. The algorithm employed for solving the nonlinear programming models is not efficient. However, for this type of problem, it seems capable of finding the solution, whereas other methods were attempted with unsatisfactory results. The time consumed by the algorithm can be improved if some adjustments are made in the values of the convergence and step-size parameters.
3. In spite of the differences between the models, it is noted that for certain variables the pattern followed is the same, even for the linear model, as in the case of borrowing and lending activities.
4. The units of projects accepted in the three models for each set of proposals A, B, and C, are different, which reflect the differences in formulation of the models. The numerical results also differ from those obtained

using the basic horizon model, indicating that the interest rate relationship does affect the capital investment decision.

In general, it can be concluded that either the non-convex model or the iterative linear model can be used to represent the relationship between borrowing interest rate and debt-equity ratio. The non-convex model includes explicit interest rate functions, and is amendable to a variety of solution techniques. However, solution times are long. The iterative linear model takes less time to solve, but not much can be said about convergence of the method. The convex model is relatively easy to solve, but its use is not recommended because of the restrictive assumption of a fixed equity throughout the planning period.

The general model is incomplete from a company's viewpoint. It is possible to improve it by including features introduced in other models: compensating balance restrictions, payback restrictions, long-term debt variables, integer solutions, payment of dividends, issuance or purchase of stock. Certainly, there would be no problem in including such features except for the last one, which requires a deeper analysis of valuation of the firm. These extensions were not considered because the main task of this study was to include an empirical relationship between the debt-equity ratio and the borrowing interest rate in a capital budgeting model, and it was not desired to include other constraints that could obstruct the analysis.

The increase in complexity of the model will likely cause a greater requirement of computer time so that other algorithms or modifications to the current one should be tested to minimize the solution time. It is also suggested that other computer algorithms be tried for solving the non-linear problem presented in this research, since the Hooke and Jeeves method consumes about 350 to 450 CPU seconds (Cyber 74) for a typical problem.

APPENDIX A

TYPICAL RESULTS FOR SET A, B, AND C GIVEN BY THE NON-CONVEX, CONVEX, AND LP MODELS

Table A-1. Results for Set A Given by the Non-Convex Nonlinear Programming Model.

Case	A.1.1	A.2.1	A.3.1	A.4.1	A.5.1	A.6.1	A.7.1	A.8.1	A.9.1	A.10.1
Initial Rate	0.075	0.125	0.150	0.175	0.225	0.100	0.250	(1)	0.0531	(2)
Obj. Function	3942	39493	39484	39493	39482	39494	39495	39489	39453	39493
r_{b0}	0.0582	0.0582	0.0581	0.0582	0.0582	0.0582	0.0582	0.0583	0.0583	0.0582
r_{b1}	0.0652	0.0652	0.0651	0.0652	0.0650	0.0652	0.0652	0.0652	0.0652	0.0652
r_{b2}	0.0708	0.0708	0.0707	0.0708	0.0703	0.0708	0.0707	0.0708	0.0709	0.0707
r_{b3}	0.0750	0.0750	0.0759	0.0708	0.0703	0.0708	0.0707	0.0708	0.0709	0.0707
r_{b4}	0.1000	0.1000	0.1022	0.1000	0.1004	0.1000	0.0999	0.1000	0.1052	0.0999
r_{b5}	0.1193	0.1193	0.1167	0.1193	0.1131	0.119	0.1186	0.1192	0.1181	0.1186
r_{b6}	0.0626	0.0532	0.0531	0.0537	0.0531	0.0532	0.0630	0.0658	0.0531	0.0531
r_{b7}	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531
x_1	0.038	0.037	0.102	0.038	0.097	0.037	0.040	0.037	0.148	0.040
x_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_3	0.670	0.670	0.014	0.650	0.598	0.659	0.654	0.659	0.598	0.654
x_4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
N	351	372	419	406	350	337	436	405	393	280
Time*	267	288	253	309	262	286	276	273	287	275

(1) and (2) are given by:

Interest Rate t	(1)	(2)
0	0.075	0.075
1	0.100	0.075
2	0.125	0.075
3	0.150	0.225
4	0.175	0.225
5	0.200	0.225
6	0.225	0.225
7	0.250	0.075

*CPU seconds for CYBER 74.

Table A-2. Results for Set B Given by the Non-Convex Nonlinear Programming Model.

Case	B.1.1	B.2.1	B.3.1	B.4.1	B.5.1	B.6.1	B.7.1
Initial Rate	0.225	0.200	0.175	0.125	0.075	0.150	0.100
Obj. Function	82694	82561	82739	82733	82738	82671	82542
r_{b0}	0.0690	0.0691	0.0691	0.0691	0.0691	0.0690	0.0690
r_{b1}	0.0760	0.0762	0.0761	0.0761	0.0761	0.0759	0.0758
r_{b2}	0.0806	0.0817	0.0812	0.0812	0.0814	0.0800	0.0794
r_{b3}	0.0889	0.0930	0.0912	0.0913	0.0919	0.0869	0.0850
r_{b4}	0.1200	0.1320	0.1263	0.1266	0.1286	0.1155	0.1201
r_{b5}	0.1183	0.1316	0.1250	0.1254	0.1276	0.1137	0.1183
r_{b6}	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531
r_{b7}	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531
x_1	0.001	0.001	0.001	0.001	0.007	0.001	0.005
x_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_4	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_5	0.000	0.000	0.000	0.000	0.004	0.000	0.000
x_6	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x_7	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_8	0.993	0.988	1.000	1.000	1.000	1.000	1.000
x_9	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x_{10}	1.000	1.000	1.000	1.000	1.000	1.000	1.000
N	492	469	399	386	337	386	459
Time*	812	769	630	637	547	638	749

*CPU seconds for CYBER 74.

Table A-3. Results for Set C Given by the Non-Convex Nonlinear Programming Model.

Case	C.1.1	C.2.1	C.3.1	C.4.1	C.5.1	C.6.1
Initial Rate	(1)	0.15	0.10	0.20	(2)	(3)
Obj. Function	83287	83312	82945	83296	83302	83325
r_{b0}	0.0779	0.0779	0.0779	0.0779	0.0779	0.0779
r_{b1}	0.0826	0.0826	0.0827	0.0826	0.0826	0.0825
r_{b2}	0.0867	0.0869	0.0867	0.0869	0.0869	0.0864
r_{b3}	0.0957	0.0964	0.0950	0.0967	0.0967	0.0945
r_{b4}	0.1223	0.1267	0.1304	0.1275	0.1274	0.1203
r_{b5}	0.1217	0.1242	0.1290	0.1250	0.1249	0.1178
r_{b6}	0.0679	0.0679	0.0679	0.0679	0.0679	0.0679
r_{b7}	0.0679	0.0679	0.0679	0.0679	0.0679	0.0679
x_1	0.018	0.019	0.015	0.020	0.020	0.016
x_2	1.000	1.000	1.000	1.000	1.000	1.000
x_3	1.000	1.000	1.000	1.000	1.000	1.000
x_4	1.000	1.000	1.000	1.000	1.000	1.000
x_5	0.000	0.000	0.000	0.000	0.000	0.000
x_6	0.000	0.000	0.028	0.000	0.000	0.000
x_7	1.000	1.000	1.000	1.000	1.000	1.000
x_8	1.000	1.000	0.997	0.997	1.000	1.000
x_9	0.000	0.000	0.000	0.000	0.000	0.000
x_{10}	1.000	1.000	1.000	1.000	1.000	1.000
N	335	336	399	354	369	368
Time*	502	402	634	525	538	555

(1), (2), and (3) are given by:

Initial Rate	(1)	(2)	(3)
t	0.15	0.400	0.075
0	0.30	0.150	0.150
1	0.10	0.400	0.225
2	0.15	0.400	0.300
3	0.30	0.125	0.300
4	0.10	0.200	0.225
5	0.15	0.350	0.150
6	0.30	0.450	0.075

*CPU seconds for CYBER 74.

Table A-4. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set A Given by the Non-Convex.

Case	t	0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	-	39492
	w_t	6697	12458	16116	18309	26646	30044	65	-
A.1.1	E_t	50000	38815	34530	31603	21476	17158	262	-
	w_t/E_t	0.1339	0.3209	0.4667	0.5793	1.2407	1.7510	0.2480	-
	IR	0.0582	0.0652	0.07084	0.0750	0.1000	0.1103	0.0626	0.0531
	v_t	-	-	-	-	-	-	-	39493
	w_t	6697	12459	16118	18308	26646	30051	70	-
A.2.1	E_t	50000	38814	34528	31605	21476	17149	21639	-
	w_t/E_t	0.1339	0.3209	0.4668	0.5792	1.2407	1.7523	0.0032	-
	IR	0.0582	0.0652	0.0708	0.0750	0.1000	0.1193	0.0532	0.0531
	v_t	-	-	-	-	-	-	152	39484
	w_t	6717	12388	16098	18791	27151	29705	-	-
A.3.1	E_t	50000	38912	34571	31090	20880	17634	-	-
	w_t/E_t	0.1343	0.3184	0.4657	0.6044	1.3003	1.6845	-	-
	IR	0.0581	0.0651	0.0707	0.0759	0.1022	0.1167	0.0531	0.0531
	v_t	-	-	-	-	-	-	-	39493
	w_t	6697	12458	16116	18309	26646	30044	65	-
A.4.1	E_t	50000	38815	34530	31603	21476	17158	33707	-
	w_t/E_t	0.1339	0.3210	0.4667	0.5793	1.2407	1.7510	0.0019	-
	IR	0.0582	0.0652	0.0708	0.0750	0.1000	0.1103	0.0532	0.0531
	v_t	-	-	-	-	-	-	510	39482
	w_t	6696	12275	15854	18559	26739	29144	-	-
A.5.1	E_t	5000	39012	34817	31325	21361	18344	-	-
	w_t/E_t	0.1339	0.3146	0.4554	0.5925	1.2518	1.5887	-	-
	IR	0.0582	0.0650	0.0703	0.0755	0.1004	0.1131	0.0531	0.0531
	v_t	-	-	-	-	-	-	-	39493
	w_t	6697	12459	16118	18308	26646	30051	70	-
A.6.1	E_t	50000	38814	34528	31605	21476	17149	21639	-
	w_t/E_t	0.1339	0.3210	0.4668	0.5793	1.2407	1.7523	0.0032	-
	IR	0.0582	0.0652	0.0708	0.0750	0.1000	0.1193	0.0532	0.0531
	v_t	-	-	-	-	-	-	-	39495
	w_t	6696	12439	16083	18308	26623	29951	4	-
A.7.1	E_t	50000	38834	34564	31603	21502	17280	14	-
	w_t/E_t	0.1339	0.3203	0.4653	0.5793	1.2387	1.7333	0.2857	-
	IR	0.0582	0.0652	0.0703	0.0750	0.0999	0.1186	0.0639	0.0531
	v_t	-	-	-	-	-	-	-	39489
	w_t	6697	12458	16115	18310	26645	30044	67	-
A.8.1	E_t	50000	38815	34530	31603	21476	17158	202	-
	w_t/E_t	0.1339	0.2492	0.4667	0.5794	1.2407	1.7510	0.3317	-
	IR	0.0583	0.0652	0.0703	0.0750	0.1000	0.1192	0.0658	0.0531
	v_t	-	-	-	-	-	-	5	39453
	w_t	6746	12427	16268	19269	27779	29910	-	-
A.9.1	E_t	5000	38901	34416	30587	20136	17396	-	-
	w_t/E_t	0.1349	0.3195	0.4727	0.6300	1.3796	1.7194	-	-
	IR	0.0583	0.0652	0.0709	0.0769	0.1052	0.1181	0.0531	0.0531
	v_t	-	-	-	-	-	-	-	39493
	w_t	6605	12440	16084	18308	26623	29956	8	-
A.10.1	E_t	50000	38833	34563	31604	21502	17274	1	-
	w_t/E_t	0.1339	0.3203	0.4654	0.5793	1.2382	1.7342	8	-
	IR	0.0582	0.0652	0.0707	0.0750	0.0990	0.1186	0.2153	0.0531

Table A-5. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set B Given by the Non-Convex Model.

Case	t	0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	5858	82694
	w_t	21087	24769	27385	31143	39930	39030	-	-
B.1.1	E_t	50000	40928	37688	32868	22215	22632	71872	-
	w_t/E_t	0.4217	0.6052	0.7266	0.9475	1.7704	1.7245	-	-
	IR	0.0690	0.0760	0.0806	0.0889	0.1200	0.1183	0.0531	0.0531
	v_t	-	-	-	-	-	-	3474	82561
	w_t	21148	24907	27994	32759	41161	41105	-	-
B.2.1	E_t	50000	40805	37029	31002	19714	19795	69423	-
	w_t/E_t	0.4230	0.6104	0.7560	1.0567	2.0879	2.0765	-	-
	IR	0.0691	0.0762	0.0817	0.0930	0.1320	0.1316	0.0531	0.0531
	v_t	-	-	-	-	-	-	4603	82739
	w_t	21129	24846	27728	32061	40356	40164	-	-
B.3.1	E_t	50000	40850	37308	31803	20833	21101	70575	-
	w_t/E_t	0.4226	0.6082	0.7432	1.0081	1.9371	1.9034	-	-
	IR	0.0691	0.0761	0.0812	0.0912	0.1263	0.1250	0.0531	0.0531
	v_t	-	-	-	-	-	-	4539	82733
	w_t	21130	24852	27745	32102	40405	40221	-	-
B.4.1	E_t	50000	40848	37293	31758	20768	21025	70511	-
	w_t/E_t	0.4226	0.6084	0.7440	1.0108	1.9455	1.9130	-	-
	IR	0.0691	0.0761	0.0812	0.0913	0.1226	0.1254	0.0531	0.0531
	v_t	-	-	-	-	-	-	6842	82738
	w_t	21094	24721	27067	30254	38490	38132	-	-
B.5.1	E_t	50000	40952	38017	33869	23315	23782	72867	-
	w_t/E_t	0.4219	0.6037	0.7120	0.8933	1.6500	1.6034	-	-
	IR	0.0690	0.0659	0.0800	0.0869	0.1155	0.1137	0.0531	0.0531
	v_t	-	-	-	-	-	-	4149	82671
	w_t	21137	24872	27838	32348	40686	40548	-	-
B.6.1	E_t	50000	40831	37192	31474	20379	20577	70114	-
	w_t/E_t	0.4227	0.6091	0.7485	1.0278	1.9965	1.9705	-	-
	IR	0.0691	0.0761	0.0814	0.0919	0.1286	0.1276	0.0531	0.0531
	v_t	-	-	-	-	-	-	8108	82542
	w_t	21052	24620	26708	29380	37330	36917	-	-
B.7.1	E_t	50000	41029	38379	34833	24777	25302	74148	-
	w_t/E_t	0.4210	0.6001	0.6959	0.8435	1.5066	1.4591	-	-
	IR	0.0690	0.0758	0.0794	0.0850	0.1201	0.1183	0.0531	0.0531

Table A-6. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set C Given by the Non-Convex Model.

Case	t	0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	806	82945
	w_t	21290	25400	28905	34174	44175	43991	-	-
C.1.1	E_t	50000	40453	36242	29732	16677	16987	67004	-
	w_t/E_t	0.4258	0.6279	0.7976	1.1494	2.6489	2.5897	-	-
	IR	0.0779	0.0827	0.0867	0.0905	0.1304	0.1290	0.0679	0.0679
	v_t	-	-	-	-	-	-	1975	83312
	w_t	21195	25235	28966	34875	43545	43115	-	-
C.2.1	E_t	50000	40525	36063	28806	17486	18076	68130	-
	w_t/E_t	0.4239	0.6227	0.8032	1.2106	2.4903	2.3852	-	-
	IR	0.0779	0.0826	0.0860	0.0964	0.1267	0.1242	0.0679	0.0679
	v_t	-	-	-	-	-	-	2500	83287
	w_t	21184	25201	28808	34477	43112	42639	-	-
C.3.1	E_t	50000	40551	36205	29259	18068	18707	68673	-
	w_t/E_t	0.4237	0.6215	0.7957	1.1783	2.3046	2.2793	-	-
	IR	0.0779	0.0826	0.0867	0.0857	0.1223	0.1217	0.0679	0.0679
	v_t	-	-	-	-	-	-	1806	83296
	w_t	21193	25235	29002	34984	43675	43260	-	-
C.4.1	E_t	50000	40525	36021	28676	17301	17876	67949	-
	w_t/E_t	0.4239	0.6227	0.8051	1.2200	2.5244	2.4200	-	-
	IR	0.0779	0.0826	0.0869	0.0967	0.1275	0.1250	0.0679	0.0679
	v_t	-	-	-	-	-	-	1822	83302
	w_t	21198	25246	29007	34980	43664	43250	-	-
C.5.1	E_t	50000	40519	36002	28688	17324	17897	67973	-
	w_t/E_t	0.4240	0.6231	0.8057	1.2193	2.5204	2.4166	-	-
	IR	0.0779	0.0826	0.0869	0.0967	0.1274	0.1249	0.0679	0.0679
	v_t	-	-	-	-	-	-	3417	83325
	w_t	21164	25142	28560	33818	42351	41803	-	-
C.6.1	E_t	50000	40596	36483	29996	19065	19686	69618	-
	w_t/E_t	0.4233	0.6193	0.7828	1.1274	2.2214	2.1128	-	-
	IR	0.0779	0.0825	0.0864	0.0945	0.1203	0.1178	0.0679	0.0679

Table A-7. Results for Set A Given by the Convex Nonlinear Programming Model.

Case	A.1.2	A.2.2	A.3.2	A.4.2	A.5.2	A.6.2	A.7.2	A.8.2	A.9.2	A.10.2
Initial Rate	0.100	0.125	0.150	0.175	0.200	0.225	0.250	(1)	(2)	(3)
Obj. Function	50530	50525	50529	50530	50530	50529	50531	50528	50529	50529
r_{b0}	0.05915	0.05920	0.05915	0.05918	0.05916	0.05918	0.05914	0.05919	0.05918	0.05914
r_{b1}	0.06634	0.06630	0.06632	0.06631	0.06634	0.06635	0.06633	0.06636	0.06632	0.06634
r_{b2}	0.07454	0.07451	0.07451	0.07454	0.07455	0.07456	0.07453	0.07456	0.07455	0.07456
r_{b3}	0.07951	0.07956	0.07952	0.07954	0.07951	0.07952	0.07953	0.07951	0.07952	0.07952
r_{b4}	0.09257	0.09257	0.09259	0.09258	0.09257	0.09258	0.09258	0.09259	0.09257	0.09257
r_{b5}	0.09134	0.09130	0.91330	0.09131	0.09132	0.09133	0.09132	0.09133	0.09136	0.09136
r_{b6}	0.05760	0.05710	0.05705	0.05703	0.05704	0.05707	0.05703	0.05701	0.05706	0.05707
r_{b7}	0.05310	0.05310	0.05310	0.05310	0.05310	0.05310	0.05310	0.05310	0.05310	0.05310
x_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
N	406	407	440	464	453	447	490	471	474	506
Time*	167	165	179	189	180	184	195	196	199	205

(1), (2), and (3) are given by:

Initial Rate t	(1)	(2)	(3)
0	0.300	0.250	0.400
1	0.175	0.225	0.075
2	0.250	0.200	0.350
3	0.125	0.175	0.225
4	0.200	0.150	0.060
5	0.075	0.125	0.250
6	0.150	0.100	0.100
7	0.053	0.075	0.150

*CPU seconds for CYBER 74.

Table A-8. Results for Set B Given by the Convex Nonlinear Programming Model.

Case	B.1.2	B.2.2	B.3.2	B.4.2	B.5.2	B.6.2	B.7.2
Initial Rate	0.125	0.075	0.100	0.150	0.200	(1)	0.225
Obj. Function	111641	111981	111642	111871	111812	111482	111641
r_{b0}	0.0732	0.0744	0.0732	0.0750	0.0735	0.0729	0.0732
r_{b1}	0.0812	0.0836	0.0812	0.0850	0.0819	0.0804	0.0812
r_{b2}	0.0912	0.0939	0.0912	0.0954	0.0920	0.0904	0.0912
r_{b3}	0.1129	0.1171	0.1129	0.1195	0.1141	0.1116	0.1129
r_{b4}	0.1299	0.1360	0.1299	0.1396	0.1317	0.1281	0.1299
r_{b5}	0.1344	0.1441	0.1344	0.1498	0.1372	0.1314	0.1344
r_{b6}	0.0813	0.0863	0.0813	0.0893	0.0827	0.0799	0.0813
r_{b7}	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531	0.0531
x_1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_2	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_4	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_5	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x_6	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_7	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_8	1.000	1.000	1.000	1.000	1.000	1.000	1.000
x_9	0.411	0.563	0.411	0.652	0.455	0.364	0.411
x_{10}	1.000	1.000	1.000	1.000	1.000	1.000	1.000
N	440	491	403	405	445	532	474
Time	436	522	418	492	451	538	500

(1) is given by:

Interest Rate t	(1)
0	0.400
1	0.075
2	0.225
3	0.300
4	0.400
5	0.150
6	0.250
7	0.100

*CPU seconds for CYBER 74.

Table A-9. Results for Set C Given by the Convex Nonlinear Programming Model.

Case	C.1.2	C.2.2	C.3.2	C.4.2	C.5.2	C.6.2
Initial Rate	(1)	(2)	(3)	0.225	0.175	0.125
Obj. Function	120292	120867	120309	120863	120524	120863
r_{b0}	0.0823	0.0832	0.0829	0.0831	0.0830	0.0831
r_{b1}	0.0913	0.0913	0.0896	0.0910	0.0909	0.0910
r_{b2}	0.0983	0.0999	0.0969	0.0981	0.0978	0.0981
r_{b3}	0.1157	0.1187	0.1141	0.1149	0.1147	0.1149
r_{b4}	0.1304	0.1336	0.1287	0.1294	0.1290	0.1294
r_{b5}	0.1406	0.1449	0.1375	0.1391	0.1386	0.1391
r_{b6}	0.0943	0.0975	0.0929	0.0934	0.0932	0.0934
r_{b7}	0.0679	0.0679	0.0679	0.0679	0.0679	0.0679
x_1	1.000	1.000	1.000	1.000	1.000	1.000
x_2	1.000	1.000	1.000	1.000	1.000	1.000
x_3	1.000	1.000	1.000	1.000	1.000	1.000
x_4	1.000	1.000	1.000	1.000	1.000	1.000
x_5	0.000	0.000	0.002	0.000	0.000	0.000
x_6	1.000	1.000	1.000	1.000	1.000	1.000
x_7	1.000	1.000	1.000	1.000	1.000	1.000
x_8	0.999	1.000	0.982	0.997	1.000	0.997
x_9	0.998	1.000	0.731	0.968	0.950	0.968
x_{10}	1.000	1.000	1.000	1.000	1.000	1.000
N	384	436	361	396	501	345
Time*	362	422	359	418	492	356

(1), (2), and (3) are given by:

Interest Rate t	(1)	(2)	(3)
0	0.0679	0.225	0.075
1	0.0679	0.225	0.075
2	0.0679	0.175	0.125
3	0.3000	0.175	0.125
4	0.3000	0.125	0.175
5	0.3000	0.125	0.175
6	0.0750	0.075	0.225
7	0.0750	0.075	0.225

*CPU seconds for CYBER 74.

Table A-10. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set A Given by the Convex Model.

Case		0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	-	50530
	w_t	7999	17472	28331	34941	52219	50555	5173	-
A.1.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3494	0.5666	0.6988	1.0443	1.0111	0.1035	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0927	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	-
	w_t	7999	17469	28323	34936	52209	50538	5166	-
A.2.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3493	0.5664	0.6087	1.0441	1.0107	0.1033	-
	IR	0.0592	0.0663	0.0745	0.0795	0.0927	0.0913	0.0571	0.0531
	v_t	-	-	-	-	-	-	-	50529
	w_t	8000	17473	28332	34943	52222	50557	5175	-
A.3.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3495	0.5666	0.6289		1.0111	0.1035	-
	IR	0.05913	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	50530
	w_t	7999	17472	28330	34940	52218	50553	5172	-
A.4.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3494	0.5666	0.6988	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.5031
	v_t	-	-	-	-	-	-	-	50530
	w_t	7999	17472	28331	34941	52219	50555	5173	-
A.5.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3494	0.5666	0.6988	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0913	0.0932	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	-
	w_t	8000	17473	28332	34943	52222	50557	5175	-
A.6.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3495	0.5666	0.6989	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	50531
	w_t	7999	17473	28331	34943	52221	50556	5173	-
A.7.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3495	0.5666	0.6789	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	50528
	w_t	8000	17473	28831	34942	52220	50556	5175	-
A.8.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3495	0.5666	0.6988	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	50529
	w_t	7999	17472	28330	34941	52219	50553	5173	-
A.9.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3494	0.5666	0.6988	1.0444	1.0111	0.1034	-
	IR	0.0591	0.0663	0.0745	0.0795	0.0985	0.0913	0.0570	0.0531
	v_t	-	-	-	-	-	-	-	50529
	w_t	8000	17473	28831	34943	52222	50557	5175	-
A.10.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.1599	0.3495	0.5666	0.6989	1.0444	1.0111	0.1034	-
	IR	0.0518	0.0663	0.0745	0.0795	0.0925	0.0913	0.0570	0.0531

Table A-11. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set B Given by the Convex Model.

Case	t	0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	-	111641
	w_t	26606	37167	50385	79087	101621	107538	37352	-
B.1.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5325	0.7433	1.0077	1.5817	2.0324	2.1508	0.7470	-
	IR	0.0732	0.0812	0.0912	0.1129	0.1299	0.1344	0.0813	0.0531
	v_t	-	-	-	-	-	-	-	111981
	w_t	28134	40361	53935	84632	109675	120363	43900	-
B.2.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5627	0.8072	1.0787	1.6926	2.1935	2.4073	0.8780	-
	IR	0.0744	0.0836	0.0939	0.1171	0.1360	0.1441	0.0863	0.0531
	v_t	-	-	-	-	-	-	-	111642
	w_t	26607	37167	50385	79087	101622	107539	37353	-
B.2.3	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5321	0.7433	1.0077	1.5817	2.0324	2.1508	0.7471	-
	IR	0.0732	0.0812	0.0912	0.1129	0.1299	0.1344	0.0813	0.0531
	v_t	-	-	-	-	-	-	-	111871
	w_t	29017	42211	55999	87861	114381	127879	47931	-
B.4.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5803	0.8442	1.1200	1.7572	2.2876	2.5576	0.9586	-
	IR	0.0750	0.0850	0.0954	0.1195	0.1396	0.1498	0.0893	0.0531
	v_t	-	-	-	-	-	-	-	111812
	w_t	27047	38084	51401	80676	103910	111196	39155	-
B.5.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5400	0.7617	1.0280	1.6135	2.0782	2.2239	0.7831	-
	IR	0.0735	0.0819	0.0920	0.1141	0.1317	0.1372	0.0827	0.0531
	v_t	-	-	-	-	-	-	-	111482
	w_t	26134	36173	49285	77376	99148	103616	35420	-
B.6.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5227	0.7235	0.9857	1.5475	1.9830	2.0723	0.7084	-
	IR	0.0729	0.0804	0.0904	0.1116	0.1281	0.1314	0.0799	0.0531
	v_t	-	-	-	-	-	-	-	111641
	w_t	26606	37167	50385	79037	101621	107538	37352	-
B.7.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.5321	0.7433	1.0077	1.5817	2.0324	2.1508	0.7470	-
	IR	0.0732	0.0812	0.0912	0.1129	0.1299	0.1344	0.0813	0.0531

Table A-12. Borrowing and Lending Activities, Equity, Debt-Equity Ratio, and Borrowing Interest Rates for Set C Given by the Convex Model.

Case	t	0	1	2	3	4	5	6	7
	v_t	-	-	-	-	-	-	-	120292
	w_t	32481	49666	64411	101195	132382	154107	55872	-
C.1.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6496	0.9933	1.2888	2.0230	2.6476	3.8210	1.1174	-
	IR	0.0823	0.0913	0.0983	0.1157	0.1304	0.1406	0.0943	0.0679
	v_t	-	-	-	-	-	-	-	120867
	w_t	32500	49705	67712	107739	139216	163131	62684	-
C.2.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6500	0.9941	1.3542	2.1548	2.7843	3.2626	1.2536	-
	IR	0.0832	0.0913	0.0999	0.1187	0.1336	0.1449	0.0975	0.0679
	v_t	-	-	-	-	-	-	-	120309
	w_t	30773	46083	61484	97823	128818	147489	52906	-
C.3.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6155	0.9217	1.2297	1.9565	2.5764	2.9498	1.0581	-
	IR	0.0824	0.0896	0.0969	0.1141	0.1287	0.1375	0.0929	0.0679
	v_t	-	-	-	-	-	-	-	120863
	w_t	32130	48973	63677	99637	130313	150912	53978	-
C.4.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6426	0.9795	1.2795	1.9927	2.6063	3.0182	1.0795	-
	IR	0.0831	0.0910	0.0981	0.1149	0.1294	0.1391	0.0934	0.0679
	v_t	-	-	-	-	-	-	-	120524
	w_t	32003	48687	63309	99161	129538	149763	53499	-
C.5.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6400	0.9737	1.2662	1.9832	2.5908	2.9953	1.0699	-
	IR	0.0830	0.0909	0.0978	0.1147	0.1290	0.1386	0.0932	0.0679
	v_t	-	-	-	-	-	-	-	120863
	w_t	32130	48973	63677	99637	130313	159082	53978	-
C.6.2	E_t	50000	50000	50000	50000	50000	50000	50000	50000
	w_t/E_t	0.6426	0.9795	1.2735	1.9927	2.6063	3.0196	1.0796	-
	IR	0.0831	0.0910	0.0981	0.1149	0.1294	0.1391	0.0934	0.0679

Table A-13. Comparison of Results for Set A Between the Basic Horizon Model, Non-Convex, Convex, and LP Models.

	1		2		3		4		5		6		7	
t	w_t	v_t	w_t	v_t	w_t	v_t	w_t	v_t	w_t	v_t	w_t	v_t	w_t	v_t
0	8000	-	8000	-	8000	-	8000	-	6696	-	7999	-	7865	-
1	17420	-	17600	-	17800	-	18000	-	12459	-	17472	-	17339	-
2	28050	-	28620	-	29280	-	29950	-	16118	-	28331	-	28006	-
3	34040	-	35270	-	36710	-	38190	-	18308	-	34941	-	34480	-
4	50350	-	52410	-	54880	-	57470	-	26646	-	52219	-	51565	-
5	46520	-	49840	-	53870	-	58150	-	30051	-	50555	-	51280	-
6	-	10090	3581	-	9253	-	15420	-	70	-	5173	-	7460	-
7	-	57050	-	52150	-	45820	-	38650	-	39493	-	50530	-	46480
x_1	1.000		1.000		1.000		1.000		0.037		1.000		0.865	
x_2	1.000		1.000		1.000		1.000		1.000		1.000		1.000	
x_3	1.000		1.000		1.000		1.000		0.670		1.000		1.000	
x_4	1.000		1.000		1.000		1.000		1.000		1.000		1.000	

1, 2, 3, 4 - Basic Horizon Model with borrowing interest rate equal to 0.0531, 0.075, 0.100, and 0.125, respectively.
5 - Non-convex Model
6 - Convex Model
7 - LP Model

APPENDIX B

FORTRAN CODE OF NON-CONVEX MODEL

```
C   * GPR *       * GPR *       * GPR *       * GPR *       * GPR *       *
      PROGRAM ABHOOKE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION D(34,34),RBIN(8),
     *EPS(8),RK(8),Q(8),QQ(8),W(8),BUFF(8),
     *PP(34),XX(34),KOUTT(7),KBB(66)
      RBIN(1)=0.10
      RBIN(2)=0.10
      RBIN(3)=0.10
      RBIN(4)=0.10
      RBIN(5)=0.10
      RBIN(6)=0.10
      RBIN(7)=0.10
      RBIN(8)=0.10
      M=34
      N=66
      NSTAGE=8
      AX=0.0679
      QD=0.
      NP=10
      NHH=16
      DO 1234 I=1,NSTAGE
 1234 RK(I)=RBIN(I)
      EPS(1)=0.025
      EPS(2)=0.025
      EPS(3)=0.025
      EPS(4)=0.01
      EPS(5)=0.06
      EPS(6)=0.05
      EPS(7)=0.025
      EPS(8)=0.025
      NCI=1
      IPRINT=1
      ITMAX=2000
      MAXK=ITMAX
      NKAT=90
      ALPHA=2.0
      BETA=0.50
      EPSY=25.0
      READ(5,1001)((D(I,J),I=1,NHH),J=1,NP)
 1001 FORMAT(F12.3)
      DO 1231 I=1,NHH
      DO 1231 J=1,NP
 1231 D(I,J)=D(I,J)/1000.
C
C
C
      WRITE(6,6001)
 6001 FORMAT(1H1,10X,"HOOKE AND JEEVES OPTIMIZATION ROUTINE",5X)
      WRITE(6,6002) ALPHA,BETA,MAXK,NKAT
 6002 FORMAT(//,2X,10HPARAMETERS,/,2X,8HALPHA = ,F5.2,4X,
     *7HBETA = ,F5.2,4X,8HITMAX = ,I4,4X,7HNKAT = ,I3)
      WRITE(6,6003) NSTAGE
 6003 FORMAT(/,2X,22HNUMBER OF VARIABLES = ,I3)
      WRITE(6,6004)
 6004 FORMAT(/,2X,18HINITIAL STEP SIZES)
      DO 6006 I=1,NSTAGE
      WRITE(6,6005) I,EPS(I)
```

```
 6005 FORMAT(/,2X,4HEPS(,I2,4H) = ,E16.8)
 6006 CONTINUE
      WRITE(6,6007) EPSY
 6007 FORMAT(/,2X,43HERROR IN FUNCTION VALUES FOR CONVERGENCE = ,E16.8)
      DO 6541 I=1,NSTAGE
      IT=I-1
      WRITE(6,6542) IT,RK(I)
 6542 FORMAT(2X,"RBIN(",I2,") = ",F8.4)
 6541 CONTINUE
      KFLAG=0
      DO 6008 I=1,NSTAGE
      Q(I)=RK(I)
      W(I)=0.
 6008 CONTINUE
      KAT=0
      KK1=0
 6070 KCOUNT=0
      WBEST=W(NSTAGE)
      DO 1985 I=1,NSTAGE
      IF(RK(I).LT.AX) RK(I)=AX
      BUFF(I)=RK(I)
 1985 CONTINUE
      CALL OBJECT(NCI,D,RK,NSTAGE,SUM,KBB,XX,PP,KOUTT)
      NCI=NCI+1
      KK1=KK1+1
      BO=SUM
      IF(KK1.EQ.1) QD=SUM
      IF(KK1.EQ.1) GO TO 6201
      IF(BO.GT.QD) KFLAG=1
      IF(BO.EQ.QD) QD=BO
C
C     ESTABLISHING THE SEARCH PATTERN
C
 6201 DO 6055 I=1,NSTAGE
      QQ(I)=RK(I)
      TSRK=RK(I)
      RK(I)=RK(I)+EPS(I)
      DO 9986 I2=1,NSTAGE
      IF(RK(I).LT.AX) RK(I)=AX
 9986 CONTINUE
      IF(NCI.GE.3) GO TO 1097
      WRITE(6,3456)
 3456 FORMAT(///,"*****************************************************",/)
      WRITE(6,3457)(RK(II),II=1,NSTAGE)
 3457 FORMAT(//,2X,8F12.3,//)
 1097 CONTINUE
      CALL OBJECT(NCI,D,RK,NSTAGE,SUM,KBB,XX,PP,KOUTT)
      NCI=NCI+1
      KK1=KK1+1
      W(I)=SUM
      IF(W(I).LT.QD) GO TO 6058
      RK(I)=RK(I)-2.*EPS(I)
      DO 9987 I22=1,NSTAGE
      IF(RK(I).LT.AX) RK(I)=AX
 9987 CONTINUE
      IF(NCI.GE.3) GO TO 3431
      WRITE(6,4987)
```

```
 4567 FORMAT(///,"00000000000000000000000000000000000000000000000000",/)
      WRITE(6,4568)(RK(LI),LI=1,NSTAGE)
 4568 FORMAT(//,2X,8F12.3,//)
 3431 CONTINUE
      CALL OBJECT(NCI,D,RK,NSTAGE,SUM,KBB,XX,PP,KOUTT)
      NCI=NCI+1
      KK1=KK1+1
      W(I)=SUM
      IF(W(I).LT.QD) GO TO 6058
      RK(I)=TSRK
      IF(I.EQ.1) GO TO 6513
      W(I)=W(I-1)
      GO TO 6613
 6513 W(I)=BO
 6613 CONTINUE
      KCOUNT=KCOUNT+1
      GO TO 6055
 6058 QD=W(I)
      QQ(I)=RK(I)
 6055 CONTINUE
      IF(IPRINT) 6060,6065,6060
 6060 WRITE(6,6100) KK1
C
C     RECORD RESPONSES AND LOCATION
C
      WRITE(6,6102)
      WRITE(6,6207)(RK(I),I=1,NSTAGE),QD
C
C     TEST TO DETERMINE TERMINATION PROGRAM
C
 6065 IF(KK1.GT.MAXK) GO TO 6094
      IF(KAT.GE.NKAT) GO TO 6094
      IF(ABS(W(NSTAGE)-WBEST).LE.EPSY) GO TO 6094
C
C     IF ALL AXES FAIL REDUCE STEP SIZE
C
      IF(KCOUNT.GE.NSTAGE) GO TO 6028
      WRITE(6,8888)
 8888 FORMAT(//,"**************************************************
     *************************")
      DO 6026 I=1,NSTAGE
      WRITE(6,7026) RK(I),BUFF(I),Q(I),QQ(I)
 7026 FORMAT(/,2X,F8.5,5X,F8.5,5X,F8.5,5X,F8.5)
      IF(Q(I).GT.RK(I)) GO TO 9753
      GO TO 9751
 9753 ALPHA=-ALPHA
 9751 RK(I)=RK(I)+ALPHA*(RK(I)-Q(I))
      IF(ALPHA.LT.0.) ALPHA=-ALPHA
      WRITE(6,7027) RK(I),ALPHA
 7027 FORMAT(//,10X,F8.5,5X,F8.3)
      WRITE(6,7028) ALPHA
 7028 FORMAT(//,10X,F8.3)
 6026 CONTINUE
      DO 6025 I=1,NSTAGE
      Q(I)=QQ(I)
 6025 CONTINUE
      GO TO 6070
```

```
      C
      C        REDUCE STEP SIZE
      C
          6028 KAT=KAT+1
               IF(KFLAG.EQ.1) GO TO 6202
               GO TO 6204
          6202 KFLAG=0
               DO  6203 I=1,NSTAGE
               RK(I)=Q(I)
          6203 CONTINUE
          6204 DO 6080 I=1,NSTAGE
               EPS(I)=EPS(I)*BETA
          6080 CONTINUE
               IF(IPRINT) 6085,6070,6085
          6085 WRITE(6,6101) KAT
               GO TO 6070
          6094 WRITE(6,6460)(EPS(I),I=1,NSTAGE)
               WRITE(6,6461)(RK(I),I=1,NSTAGE)
               WRITE(6,6462) QQ
               DO 6104 I=1,NSTAGE
          6104 WRITE(6,6103) I,RK(I)
               WRITE(6,6100) KK1
          6100 FORMAT(//,2X,33HNUMBER OF FUNCTION EVALUATIONS = ,I8)
          6101 FORMAT(/,2X,18HSTEP SIZE REDUCED ,I2,6H TIMES)
          6102 FORMAT(1X,26HEND OF EACH PATTERN SEARCH/)
          6103 FORMAT(//,2X,8HFINAL X(,I2,4H) = ,E16.8)
          6207 FORMAT(1X,"VARIABLES AND SUMN",3X,9E12.4//)
          6465 FORMAT(10X,"SUM",3X,E14.5)
          6460 FORMAT(1X,"THE FINAL EPS ARE ",9F12.8/)
          6461 FORMAT(1X,"THE FINAL RK ARE ",8F12.8/)
          6462 FORMAT(1X,"THE MINIMUM RESPONSE IS ",F20.3/)
               CALL OBJECT(NCI,D,RK,NSTAGE,SUM,KBB,XX,PP,KOUTT)
               WRITE(6,903)(KOUTT(I),I=1,7)
               WRITE(6,905)
               WRITE(6,900)(KBB(I),I=1,N)
               WRITE(6,905)
               WRITE(6,901)(XX(I),I=1,M)
               WRITE(6,905)
               WRITE(6,901)(PP(I),I=1,M)
           900 FORMAT(10I5)
           901 FORMAT(10F12.3)
           903 FORMAT(7I5)
           905 FORMAT(//)
               STOP
               END
```

SYMBOLIC REFERENCE MAP (R=1)

RY POINTS
37   ABHOOKE

SUBROUTINE OBJECT 74/74 OPT=1 FTN 4.6+460

```
      C
      C
      C
      C
      C
            SUBROUTINE OBJECT(NCI,D,RK,NSTAGE,SUM,KB3,XX,PP,KOUTT)
            DIMENSION A(34,66),B(34),TOL(4),E(34,34),KB(66),JH(34),
           1INFIX(8),X(34),P(34),Y(34),XX(34),KB3(66),PP(34),
           1KOUT(8),ERR(8),D(34,34),RK(8),KOUTT(7)
            MVA=1
            EQ=50000.
            NHH=16
            M=34
            N=66
            NH=9
            NP=10
            AX=0.0679
            BX=0.023595
            DO 7777 L=1,M
            DO 7777 LL=1,N
       7777 A(L,LL)=0.0
            DO 7771 M1=1,NHH
            DO 7771 M2=1,NP
       7771 A(M1,M2)=D(M1,M2)
            TOL(1)=0.00001
            TOL(2)=0.00001
            TOL(3)=-0.001
            TOL(4)=0.0000000001
            INFIX(1)=4
            INFIX(2)=66
            INFIX(3)=34
            INFIX(4)=34
            INFIX(5)=2
            INFIX(6)=1
            INFIX(7)=999
            INFIX(8)=0
            PRM=0.
            A(1,18)=-1.
            A(1,26)=1.
            I=2
            J=11
         55 A(I,J)=1.
            I=I+1
            J=J+1
            IF(I.GT.9) GO TO 5
            GO TO 55
          5 I=2
            J=19
         44 A(I,J)=-1.
            I=I+1
            J=J+1
            IF(I.GT.9) GO TO 6
            GO TO 44
          6 I=2
            J=34
         66 A(I,J)=1.
            I=I+1
```

```
            J=J+1
            IF(I.GT.34) GO TO 7
            GO TO 66
          7 I=10
            J=12
         77 A(I,J)=-0.055
            I=I+1
            J=J+1
            IF(I.GT.16) GO TO 8
            GO TO 77
          8 I=10
            J=27
         88 A(I,J)=1.
            I=I+1
            J=J+1
            IF(I.GT.16) GO TO 10
            GO TO 88
         10 I=25
            J=1
       1010 A(I,J)=0.001
            I=I+1
            J=J+1
            IF(I.GT.34) GO TO 11
            GO TO 1010
         11 I=3
            J=11
       9999 A(I,J)=-1.055
            I=I+1
            J=J+1
            IF(I.GT.9) GO TO 12
            GO TO 9999
         12 I=17
            J=19
       1313 A(I,J)=BX
            I=I+1
            J=J+1
            IF(I.GT.24) GO TO 13
            GO TO 1313
         13 I=11
            J=27
       1312 A(I,J)=-1.
            I=I+1
            J=J+1
            IF(I.GT.16) GO TO 14
            GO TO 1312
         14 CONTINUE
            DO 303 I=3,9
            A(I,I+15)=RK(I-2)+1.
        303 A(I+7,I+17)=RK(I-1)
            DO 311 LL=18,24
            A(LL,LL+9)=-RK(LL-15)+AX
        311 CONTINUE
            B(1)=0.
            B(2)=1000.
            B(3)=5000.
            B(4)=300.
            B(5)=500.
```

```
      B(6)=500.
      B(7)=0.
      B(8)=500.
      B(9)=1000.
      B(10)=EQ
      IF(MVA.EQ.1) GO TO 4444
      B(11)=300.
      B(12)=500.
      B(13)=500.
      B(14)=0.
      B(15)=500.
      B(16)=1000.
      GO TO 433
 4444 CONTINUE
      DO 4445 I=11,16
 4445 B(I)=0.
  433 B(17)=EQ*(RK(1)-AX)
      DO 204 K=25,34
  204 B(K)=1.
      DO 205 I=18,24
  205 B(I)=0.
      IF(NCI.GE.3) GO TO 1234
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=1,10),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=11,20),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=21,30),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=31,40),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=41,50),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=51,60),I=1,M)
      WRITE(6,105)
      WRITE(6,100)((A(I,J),J=61,65),I=1,M)
      WRITE(6,105)
      WRITE(6,103)(B(I),I=1,M)
      WRITE(6,105)
  103 FORMAT(F12.3)
  105 FORMAT(1H1,/////)
  106 FORMAT(10F12.3)
  100 FORMAT(6F12.3)
 1234 CONTINUE
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,K3,E)
      SUM=-X(1)
      DO 107 K=1,N
  107 K3B(K)=K3(K)
      DO 108 K=1,M
      PP(K)=P(K)
  108 XX(K)=X(K)
      DO 1108 I=1,7
 1108 KOUTT(I)=KOUT(I)
      IF(NCI.LE.300) GO TO 6177
      WRITE(6,202)
      WRITE(6,203)(KB3(I),I=1,N)
      WRITE(6,202)
```

```
SUBROUTINE OBJECT      74/74    OPT=1                                  FTN 4.6+460

                              WRITE(6,201)(XX(I),I=1,M)
                              WRITE(6,202)
                              WRITE(6,201)(PP(I),I=1,M)
                              WRITE(6,202)
                              WRITE(6,207)(KOUT(I),I=1,8)
                              WRITE(6,202)
                          201 FORMAT(10F12.3)
                          202 FORMAT(/////)
                          203 FORMAT(15X,10I5)
                          207 FORMAT(15X,8I5)
                         6177 CONTINUE
                              DO 109 K=1,M
                              P(K)=0.
                              X(K)=0.
                              Y(K)=0.
                          109 JH(K)=0.
                              DO 110 K=1,M
                              DO 110 KK=1,M
                          110 E(K,KK)=0.
                              DO 111 K=1,7
                          111 KOUT(K)=0
                              DO 112 K=1,8
                          112 ERR(K)=0.
                              DO 113 I=1,NHH
                              DO 113 J=11,N
                          113 A(I,J)=0.
                              DO 114 I=17,M
                              DO 114 J=1,N
                          114 A(I,J)=0.
                              DO 1144 I=1,M
                         1144 B(I)=0.
                              DO 115 I=1,N
                          115 KB(I)=0
                              RETURN
                              END
```

```
     SYMBOLIC REFERENCE MAP (R=1)

TRY POINTS
  4  OBJECT

RIABLES        SN  TYPE             RELOCATION
305  A             REAL        ARRAY                       1272  AX       REAL
611  B             REAL        ARRAY                       1273  BX       REAL
  J  D             REAL        ARRAY       F.P.            5657  E        REAL          ARRAY
264  EQ            REAL                                   10415  ERR      REAL          ARRAY
301  I             INTEGER                                10227  INFIX    INTEGER       ARRAY
302  J             INTEGER                                10165  JH       INTEGER       ARRAY
303  K             INTEGER                                10063  KB       INTEGER       ARRAY
  3  KBB           INTEGER     ARRAY       F.P.            1304  KK       INTEGER
405  KOUT          INTEGER     ARRAY                          0  KOUTT    INTEGER       ARRAY
274  L             INTEGER                                 1275  LL       INTEGER
266  M             INTEGER                                 1263  MVA      INTEGER
```

APPENDIX C

FORTRAN CODE OF CONVEX MODEL

```
C B11R    B11R    B11R    B11R    B11R    B11R    B11R    B11R
      PROGRAM ABHCCKE(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION TCL(4),
     +X(27),F(27),Y(27),XX(27),KEB(52),PP(27),
     +EPR(8),D(27,27),RBIN(8),
     +EPS(8),RK(8),Q(8),QQ(8),W(8),BUFF(8)
C
C       ITMAX=MAXIMUM NUMBER OF TIMES THE OBJECTIVE FUNCTION IS CALLE
C
C       NKAT=MAXIMUM NUMBER OF TIMES THE INITIAL STEP SIZE IS TO BE
C       REDUCED
C
C       NSTAGE=NUMBER OF DECISION VARIBLES TO BE USED
C
      READ(5,5002) ITMAX,NKAT,NSTAGE
 5002 FORMAT(I3,I3,I3)
C
C
C
C
C       RBIN=VECTOR OF THE STARTING SOLUTION
C
      READ(5,5000)(RBIN(I),I=1,NSTAGE)
 5000 FORMAT(12.5)
C
C
C
C
C       EPS=VECTOR OF THE INITIAL STEP SIZE TO BE USED FOR
C       EACH OF THE VARIABLES
C
      READ(5,5001)(EPS(I),I=1,NSTAGE)
 5001 FORMAT(12.5)
C
C
C
C
C       ALPHA=FACTOR FOR EXTENDING THE SIZE OF THE INITIA
C       STEPS, GREATER THAN OR EQUAL TO 1.0
C
C       BETA=FACTOR FOR REDUCING THE INITIAL STEP SIZE,
C       0<=BETA<=1.
C
C       EPSY=ERROR IN OBJECTIVE FUNCTION TO BE REACHED BEFORE
C       PROGRAM TERMINATES (DIFFERENCE BETWEEN CURRENT VALUE
C       AND PREVIOUS STAGE VALUE).
C
      READ(5,5003) ALPHA,BETA,EPSY
 5003 FORMAT(F5.2,F5.2,F6.3)
C
C
C
C       AX=INTERCEPT OF THE INTEREST RATE-DEBT/EQUITY RATIO
C       REGRESSION EQUATION
C
C       BX=SLOPE OF THE INTEREST RATE-DEBT/EQUITY RATIO
C       REGRESSION EQUATION
C
C       AR=LENDING INTEREST RATE
C
```

```
      READ(5,5004) AX,BX,AR
 5004 FORMAT(3(F12.5))
C
C
C
C        EQ=INITIAL EQUITY
C
      READ(5,5005) EQ
 5005 FORMAT(12.5)
C
C
C
C        M=NUMBER OF CONSTRAINTS IN THE CAPITAL BUDGETING
C        MODEL + 1
C
C        NP=NUMBER OF PROJECTS
C
C        NH=NUMBER OF CASH BALANCE CONSTRAINTS + NUMBER OF
C        EQUITY CONSTRAINTS
C
C        N=NUMBER OF VARIABLES + NUMBER OF SLACKS + 1
C
      READ(5,5006) M,NP,NH,N
 5006 FORMAT(4(I5))
C
C
C
      CC=0.
      DO 1234 I=1,8
 1234 RK(I)=RBIN(I)
      NCI=1
      IPRINT=1
      MAXK=ITMAX
      READ(5,1001)((D(I,J),I=1,NH),J=1,NP)
 1001 FORMAT(F12.3)
C
C
C
      WRITE(6,6001)
 6001 FORMAT(1H1,10X,"HOOKE AND JEEVES OPTIMIZATION ROUTINE",5X)
      WRITE(6,6002) ALPHA,BETA,MAXK,NKAT
 6002 FORMAT(//,2X,10HPARAMETERS,/,2X,8HALPHA = ,F5.2,4X,
     *7HBETA = ,F5.2,4X,8HITMAX = ,I4,4X,7HNKAT = ,I3)
      WRITE(6,6003) NSTAGE
 6003 FORMAT(/,2X,22HNUMBER OF VARIABLES = ,I3)
      WRITE(6,6004)
 6004 FORMAT(/,2X,18HINITIAL STEP SIZES)
      DO 6006 I=1,NSTAGE
      WRITE(6,6005) I,EPS(I)
 6005 FORMAT(/,2X,4HEPS(,I2,4H) = ,E16.8)
 6006 CONTINUE
      WRITE(6,6007) EPSY
 6007 FORMAT(/,2X,43HERROR IN FUNCTION VALUES FOR CONVERGENCE = ,E1
      DO 6541 I=1,NSTAGE
      IT=I-1
      WRITE(6,6542) IT,RK(I)
 6542 FORMAT(2X,"REIN(",I2,") = ",F8.4)
```

```
      6541 CONTINUE
           KFLAG=0
           DO 6008 I=1,NSTAGE
           Q(I)=RK(I)
           W(I)=0.
      6008 CONTINUE
           KAT=0
           KK1=0
      6070 KCOUNT=0
           WBEST=W(NSTAGE)
           DO 1985 I=1,NSTAGE
           IF(RK(I).LT.AX) RK(I)=AX
      1985 CONTINUE
           CALL CEJECT(AR,AX,BX,NCI,D,RK,NSTAGE,SUM,EQ,M,NP,NH,N)
           KK1=KK1+1
           BO=SUM
           IF(KK1.EQ.1) QD=SUM
           IF(KK1.EQ.1) GO TO 6201
           IF(BO.GT.QD) KFLAG=1
           IF(BO.EQ.QD) QD=BO
C
C          ESTABLISHING THE SEARCH PATTERN
C
      6201 DO 6055 I=1,NSTAGE
           QQ(I)=RK(I)
           TSRK=RK(I)
           RK(I)=RK(I)+EPS(I)
           DO 9986 I2=1,NSTAGE
           IF(RK(I).LT.AX) RK(I)=AX
      9986 CONTINUE
           CALL CEJECT(AR,AX,BX,NCI,D,RK,NSTAGE,SUM,EQ,M,NP,NH,N)
           KK1=KK1+1
           W(I)=SUM
           IF(W(I).LT.QD) GO TO 6058
           RK(I)=RK(I)-2.*EPS(I)
           DO 9987 I22=1,NSTAGE
           IF(RK(I).LT.AX) RK(I)=AX
      9987 CONTINUE
           WRITE(6,4568)(RK(LI),LI=1,NSTAGE)
      4568 FORMAT(//,2X,8F12.3,//)
           CALL CEJECT(AR,AX,BX,NCI,D,RK,NSTAGE,SUM,EQ,M,NP,NH,N)
           NCI=NCI+1
           KK1=KK1+1
           W(I)=SUM
           IF(W(I).LT.QD) GO TO 6058
           RK(I)=TSRK
           IF(I.EQ.1) GO TO 6513
           W(I)=W(I-1)
           GO TO 6613
      6513 W(I)=BO
      6613 CONTINUE
           KCOUNT=KCOUNT+1
           GO TO 6055
      6058 QD=W(I)
           QQ(I)=RK(I)
      6055 CONTINUE
           IF(IPRINT) 6060,6065,6060
```

```
      6060 WRITE(6,6100) KK1
C
C          RECORD RESPONSES AND LOCATION
C
           WRITE(6,6102)
           WRITE(6,6207)(RK(I),I=1,NSTAGE),QD
C
C          TEST TO DETERMINE TERMINATION PROGRAM
C
      6065 IF(KK1.GT.MAXK) GO TO 6094
           IF(KAT.GE.NKAT) GO TO 6094
           IF(ABS(W(NSTAGE)-WBEST).LE.EPSY) GO TO 6094
C
C          IF ALL AXES FAIL REDUCE STEP SIZE
C
           IF(KCOUNT.GE.NSTAGE) GO TO 6028
           DO 6026 I=1,NSTAGE
           RK(I)=RK(I)+ALPHA*(RK(I)-Q(I))
      6026 CONTINUE
           DO 6025 I=1,NSTAGE
           Q(I)=CC(I)
      6025 CONTINUE
           GO TO 6070
C
C          REDUCE STEP SIZE
C
      6028 KAT=KAT+1
           IF(KFLAG.EQ.1) GO TO 6202
           GO TO 6204
      6202 KFLAG=0
           DO  6203 I=1,NSTAGE
           RK(I)=C(I)
      6203 CONTINUE
      6204 DO 6080 I=1,NSTAGE
           EPS(I)=EPS(I)*BETA
      6080 CONTINUE
           IF(IPRINT) 6085,6070,6085
      6085 WRITE(6,6101) KAT
           GO TO 6070
      6094 WRITE(6,6460)(EPS(I),I=1,NSTAGE)
           WRITE(6,6461)(RK(I),I=1,NSTAGE)
           WRITE(6,6462) QD
           DO 6104 I=1,NSTAGE
      6104 WRITE(6,6103) I,RK(I)
           WRITE(6,6100) KK1
      6100 FORMAT(//,2X,33HNUMBER OF FUNCTION EVALUATIONS = ,I8)
      6101 FORMAT(/,2X,18HSTEP SIZE REDUCED ,I2,6H TIMES)
      6102 FORMAT(1X,26HEND OF EACH PATTERN SEARCH/)
      6103 FORMAT(//,2X,3HFINAL X(,I2,4H) = ,E16.8)
      6207 FORMAT(1X,"VARIABLES AND SUMN",3X,9E12.4//)
      6465 FORMAT(10X,"SUM",3X,E14.5)
      6460 FORMAT(1X,"THE FINAL EPS ARE ",8F12.8/)
      6461 FORMAT(1X,"THE FINAL RK ARE ",8F12.8/)
      6462 FORMAT(1X,"THE MINIMUM RESPONSE IS ",F20.8/)
           NCI=0
           CALL CEJECT(AR,AX,BX,NCI,D,RK,NSTAGE,SUM,EQ,M,NP,NH,N)
           STOP
```

```
      C
      C
                SUBROUTINE OBJECT(AR,AX,EX,NCI,D,RK,NSTAGE,SUM,M,NP,NH,N)
                DIMENSION A(27,52),B(27),TOL(4),E(27,27),KB(52),JH(27),
               *INFIX(8),X(27),P(27),Y(27),XX(27),KBB(52),PP(27),
               *KOUT(7),ERR(8),D(27,27),
               *K(8)
                DO 7777 I=1,M
                DO 7777 J=1,N
           7777 A(I,J)=0.
                DO 7771 M1=1,NH
                DO 7771 M2=1,NP
           7771 A(M1,M2)=D(M1,M2)
      C
      C
      C
      C          TOL(1)=PIVOT TOLERANCE
      C          TOL(2)=TOLERANCE FOR SETTING "X" TO ZERO
      C          TOL(3)=REDUCED COST IS CONSIDERED TO BE NEGATIVE ONLY IF
      C          IS BELOW THIS QUANTITY
      C          TOL(4)=QUANTITIES IN THE PIVOT ROW OF THE INVERESE ARE
      C          ASSUMED ZERO IF MAGNITUDE BELOW THIS QUANTITY
      C
      C
      C
                TOL(1)=0.00001
                TOL(2)=0.00001
                TOL(3)=-0.001
                TOL(4)=0.0000000001
      C
      C
      C
      C          INFIX(1)=INFLAG
      C
      C          INFIX(2)=N
      C
      C          INFIX(3)=M
      C
      C          INFIX(4)=M
      C
      C          INFIX(5)=THE ROW NUMBER OF THE FIRST CONSTRAINT IN
      C
                INFIX(1)=4
                INFIX(2)=N
                INFIX(3)=M
                INFIX(4)=M
                INFIX(5)=2
                INFIX(6)=1
                INFIX(7)=999
                INFIX(8)=0
                PRM=0.
                A(1,18)=-1.
                A(1,26)=1.
                I=2
                J=11
             55 A(I,J)=1.
                I=I+1
```

```
      J=J+1
      IF(I.GT.9) GO TO 5
      GO TO 55
    5 I=2
      J=19
   44 A(I,J)=-1.
      I=I+1
      J=J+1
      IF(I.GT.9) GO TO 6
      GO TO 44
    6 I=2
      J=27
   66 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.9) GO TO 7
      GO TO 66
    7 I=10
      J=19
   77 A(I,J)=BX
      I=I+1
      J=J+1
      IF(I.GT.17) GO TO 8
      GO TO 77
    8 I=10
      J=35
   88 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.17) GO TO 9
      GO TO 88
    9 I=18
      J=1
   99 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.27) GO TO 10
      GO TO 99
   10 I=18
      J=43
 1010 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.27) GO TO 11
      GO TO 1010
   11 I=3
      J=11
 9999 A(I,J)=-(1.+AR)
      I=I+1
      J=J+1
      IF(I.GT.9) GO TO 12
      GO TO 9999
   12 DO 333 I=3,9
  333 A(1,I+16)=FK(I-2)+1.
      B(1)=0.
      B(2)=1000.
      B(3)=5000.
```

```
      B(4)=300.
      B(5)=500.
      B(6)=500.
      B(7)=0.
      B(8)=500.
      B(9)=1000.
      DO 201 K=10,17
  201 B(K)=RK(K-9)*EC-EQ*AX
      DO 204 K=18,27
  204 B(K)=1.
      WRITE(6,8105)(RK(I),I=1,NSTAGE)
 8105 FORMAT(//,2X,8F12.3,//)
      IF(NCI.NE.0) GO TO 1928
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=1,10),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=11,20),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=21,30),I=1,M)
      WRITE(6,105)
      WRITE(6,106) ((A(I,J),J=31,40),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=41,50),I=1,M)
      WRITE(6,105)
      WRITE(6,107)((A(I,J),J=51,52),I=1,M)
      WRITE(6,105)
      WRITE(6,103)(B(I),I=1,M)
      WRITE(6,105)
  103 FORMAT(F12.3)
  105 FORMAT(1H1,/////)
  106 FORMAT(10F12.3)
  107 FORMAT(2F12.3)
 1928 CONTINUE
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E)
      SUM=-X(1)
      DO 1070 K=1,N
 1070 KBB(K)=KB(K)
      DO 108 K=1,M
      FP(K)=F(K)
  108 XX(K)=X(K)
      IF(NCI.NE.0) GO TO 5055
      WRITE(6,1051)(KBB(K),K=1,N)
 1051 FORMAT(12X,10I5)
      WRITE(6,1052)
      WRITE(6,1053)(FP(I),I=1,M)
 1053 FORMAT(10F12.3)
      WRITE(6,1052)
      WRITE(6,1054)(XX(L1),L1=1,M)
      WRITE(6,1052)
      WRITE(6,1060)(KOUT(I),I=1,7)
 1060 FORMAT(5X,I2,3X,I2,3X,I2,3X,I2,3X,I2,3X,I2,3X,I2)
      WRITE(6,1052)
 1054 FORMAT(10F12.3)
 1052 FORMAT(/////)
 5055 CONTINUE
      DO 109 K=1,M
      P(K)=0.
      Y(K)=0.
  109 JH(K)=0.
      DO 110 K=1,M
      DO 110 KK=1,M
  110 E(K,KK)=0.
      DO 111 K=1,7
  111 KOUT(K)=0.
      DO 112 K=1,8
  112 ERR(K)=0.
      DO 113 I=1,NM
      DO 113 J=11,N
  113 A(I,J)=0.
      DO 114 I=10,M
      DO 114 J=1,N
  114 A(I,J)=0.
      DO 1144 I=1,M
 1144 P(I)=0.
      RETURN
      END
```

APPENDIX D

FORTRAN CODE OF LP MODEL

```
C  *   GPA   *   GPA   *
C  *   GPA   *   GPA   *   GPA   *   GPA   *
C  *   GPA   *   GPA   *   GPA   *   GPA   *
C
C
C
C              GPA USES AVV (DATA FILE) ******************************
C              ************************************************************
C
C
C
C
      PROGRAM AR(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION A(20,46),B(20),TOL(4),E(20,20),KB(46),JH(20),
     1INFIX(8),X(20),P(20),Y(20),XX(20),KBB(46),PP(20),
     1KOUT(7),ERR(8),D(20,20)
      IC=1
      EQ=50000.
      NHH=16
      S2=0.0
      M=20
      N=47
      NH=9
      NP=4
      KKK=1
      RBIN=0.15
      AX=0.0531
      BX=0.0378
      DO 7777 L=1,M
      DO 7777 LL=1,N
 7777 A(L,LL)=0.0
      READ(5,1001)((D(I,J),I=1,NHH),J=1,NP)
 1001 FORMAT(F12.3)
      DO 7771 M1=1,NHH
      DO 7771 M2=1,NP
 7771 A(M1,M2)=D(M1,M2)/1000.
      TOL(1)=0.00001
      TOL(2)=0.00001
      TOL(3)=-0.001
      TOL(4)=0.000000001
      INFIX(1)=4
      INFIX(2)=46
      INFIX(3)=20
      INFIX(4)=20
      INFIX(5)=2
      INFIX(6)=1
      INFIX(7)=999
      INFIX(8)=0
      PRM=0.
 5000 A(1,12)=-1.
      A(1,20)=1.
      I=2
      J=5
   55 A(I,J)=1.0
      I=I+1
      J=J+1
      IF(I.GT.9) GO TO 5
```

```
      GO TO 55
    5 I=2
      J=13
   44 A(I,J)=-1.
      I=I+1
      J=J+1
      IF(I.GT.16) GO TO 6
      GO TO 44
    6 I=2
      J=28
   66 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.20) GO TO 7
      GO TO 66
    7 I=10
      J=6
   77 A(I,J)=-0.04
      I=I+1
      J=J+1
      IF(I.GT.16) GO TO 8
      GO TO 77
    8 I=10
      J=21
   88 A(I,J)=1.
      I=I+1
      J=J+1
      IF(I.GT.16) GO TO 9
      GO TO 88
    9 I=17
      J=1
 1010 A(I,J)=0.001
      I=I+1
      J=J+1
      IF(I.GT.20) GO TO 11
      GO TO 1010
   11 I=3
      J=5
 9999 A(I,J)=-1.04
      I=I+1
      J=J+1
      IF(I.GT.9) GO TO 12
      GO TO 9999
   12 I=11
      J=21
 1414 A(I,J)=-1.0
      I=I+1
      J=J+1
      IF(I.GT.16) GO TO 1313
      GO TO 1414
 1313 B(10)=EQ
      DO 303 I=3,9
      A(I,I+10)=RBIN+1.
  303 A(I+7,I+11)=RBIN
      B(1)=0.
      B(2)=1000.
      B(3)=5000.
```

```
      B(4)=300.
      B(5)=500.
      B(6)=500.
      B(7)=0.
      B(8)=500.
      B(9)=1000.
      DO 4445 I=11,16
 4445 B(I)=0.
 4443 CONTINUE
      DO 204 K=17,20
  204 B(K)=1.
      IF(KKK.NE.1) GO TO 2344
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=1,10),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=11,20),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=21,30),I=1,M)
      WRITE(6,105)
      WRITE(6,106)((A(I,J),J=31,40),I=1,M)
      WRITE(6,105)
      WRITE(6,100)((A(I,J),J=41,46),I=1,M)
      WRITE(6,105)
      WRITE(6,103)(B(I),I=1,M)
      WRITE(6,105)
  103 FORMAT(F12.3)
  105 FORMAT(1H1,/////)
  106 FORMAT(10F12.3)
  100 FORMAT(6F12.3)
 2344 CONTINUE
      CALL SIMPLX(INFIX,A,B,TOL,PRM,KOUT,ERR,JH,X,P,Y,KB,E)
      S1=-X(12)+X(20)
      DO 107 K=1,N
  107 KBB(K)=KB(K)
      DO 108 K=1,M
      PP(K)=P(K)
  108 XX(K)=X(K)
      WRITE(6,202)
      WRITE(6,203)(KBB(I),I=1,N)
      WRITE(6,202)
      WRITE(6,201)(XX(I),I=1,M)
      WRITE(6,202)
      WRITE(6,201)(PP(I),I=1,M)
      WRITE(6,202)
      WRITE(6,207)(KOUT(I),I=1,7)
      WRITE(6,202)
  201 FORMAT(10F12.3)
  202 FORMAT(/////)
  203 FORMAT(15X,10I5)
  207 FORMAT(15X,8I5)
      WRITE(6,225)
  225 FORMAT(1H1,////////////////)
      DO 109 K=1,M
      P(K)=0.
      Y(K)=0.
  109 JH(K)=0.
      DO 110 K=1,M
```

```
      DO 110 KK=1,M
  110 E(K,KK)=0.
      DO 111 K=1,7
  111 KOUT(K)=0.
      DO 112 K=1,8
  112 ERR(K)=0.
      DO 113 I=1,NHH
      DO 113 J=5,N
  113 A(I,J)=0.
      DO 114 I=17,M
      DO 114 J=1,N
  114 A(I,J)=0.
      DO 1144 I=1,M
 1144 B(I)=0.
      DO 115 I=1,N
  115 KB(I)=0.
      EQU=50000.
      DEU=0.
      DO 2817 I=13,20
      MW=KBE(I)
      IF(MW.EQ.0) GO TO 2818
      DEU=DEU+XX(MW)
      GO TO 2817
 2818 DEU=DEU
 2817 CONTINUE
      DO 2815 IX=21,27
      ME=KBE(IX)
      IF(ME.EQ.0) GO TO 2820
      EQU=EQU+XY(ME)
      GO TO 2815
 2820 EQU=EQU
 2815 CONTINUE
      DERA=DEU/EQU
      RBIN=AX+BX*DERA
      WRITE(6,2821) DERA,RBIN,IC
 2821 FORMAT(/,10X,F12.3,5X,F12.3,5X,I5,/)
      IC=IC+1
      WRITE(6,1236)
 1236 FORMAT(/////)
      LOOM=LOOM+1
      KKK=KKK+1
      AS=ABS(S1-S2)
      IF(AS.LE.2.) GO TO 5001
      S2=S1
      GO TO 5000
 5001 CONTINUE
      STOP
      END
```

SYMBOLIC REFERENCE MAP (R=1)

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